Journal of Rajasthan Academy of Physical Sciences
ISSN : 0972-6306; URL : http://raops.org.in
Vol.18, No.1&2, January-June, 2019, 1-14

CERTAIN RELATION OF GENERALIZED FRACTIONAL CALCULUS ASSOCIATED WITH THE PRODUCT OF GENERALIZED MITTAG-LEFFLER FUNCTION AND SRIVASTAVA POLYNOMIALS

Bhupender Singh Shaktawat[1], Madan Lal[2] and **Rajeev Kumar Gupta[2]**
[1]Department of Mathematics, Dayanand College, Ajmer-305001, India.
[2]Department of Mathematics and Statistics, Jai Narain Vyas University, Jodhpur-342005, India.
E-mail: b.s.shaktawatmath@gmail.com, madan.lakhani288@gmail.com
drrkbgupta@yahoo.co.in

Abstract: The aim of this paper is to establish four image formulas for generalized fractional integral and derivative operators, applied on the product of generalized Mittag-Leffler function and Srivastava polynomials. The results are expressed in terms of generalized Wright function. Recent results of Gupta et al., Gupta and Parihar, Saxena and Saigo are obtained as special cases of our main findings.

Keywords and phrases: Generalized fractional calculus operators, Generalized Mittag-Leffler function, Srivastava polynomials, generalized Wright function and special functions.

Mathematics subject classification: 26A33, 33B15, 33C45, 33C70, 33E12.

1. Introduction

In 1903, the function $E_\vartheta(z)$ was introduced by the Swedish mathematician Gosta Mittag-Leffler [4], and defined as

$$E_\vartheta(z)=\sum_{n=0}^{\infty}\frac{z^n}{\Gamma(\vartheta n+1)},\left(\vartheta\in C,\Re(\vartheta)>0,z\in C\right), \tag{1}$$

the Mittag-Leffler function (1) is a direct generalization of $exp(z)$ in which $\vartheta=1$.

A generalization of $E_\vartheta(z)$ was studied by Wiman [13], given by

$$E_{\vartheta,\rho}(z)=\sum_{n=0}^{\infty}\frac{z^n}{\Gamma(\vartheta n+\rho)},\left(\vartheta,\rho\in C,\Re(\vartheta)>0,\Re(\rho)>0,z\in C\right). \tag{2}$$

Further generalization of Mittag-Leffler function was introduced by Prabhakar [5] in the following form:

$$E_{\vartheta,\rho}^{\delta}(z)=\sum_{n=0}^{\infty}\frac{(\delta)_n z^n}{\Gamma(\vartheta n+\rho)n!}, \tag{3}$$

where $\vartheta,\rho,\delta,z\in C,\Re(\vartheta)>0,\Re(\rho)>0,\Re(\delta)>0$ and

$$(\delta)_n=\frac{\Gamma(\delta+n)}{\Gamma(\delta)}, \tag{4}$$

here, $(\delta)_n$ denotes the Pochhammer symbol.

Recently, a new generalization of Mittag-Leffler function introduced by Salim and Faraj [8] in the following manner:

$$E_{\alpha,\beta,q}^{\delta,\xi,p}(z)=\sum_{n=0}^{\infty}\frac{(\delta)_{qn} z^n}{\Gamma(\alpha n+\beta)(\xi)_{pn}}, \tag{5}$$

where, $\alpha,\beta,\delta,\xi\in C,\Re(\alpha),\Re(\beta),\Re(\delta),\Re(\xi)>0;\ p,q>0,\ q\le\Re(\alpha)+p$ and

$$(\gamma)_{qn}=\frac{\Gamma(\gamma+qn)}{\Gamma(\gamma)}, \tag{6}$$

here, $(\gamma)_{qn}$ denotes the generalized Pochhammer symbol.

Remark 1: If we take $\xi=p=q=1$ and $\delta=\xi=p=q=1$, equation (5) reduces to generalized Mittag-Leffler function $E_{\alpha,\beta}^{\delta}(z)$ and $E_{\alpha,\beta}(z)$ respectively.

The generalized Wright hypergeometric function introduced by Wright [14] as:

$${}_p\psi_q(z)={}_p\psi_q\left[\begin{matrix}(a_i,A_i)_{1,p}\\(b_j,B_j)_{1,q}\end{matrix};z\right]={}_p\psi_q\left[\begin{matrix}(a_1,A_1),\dots,(a_p,A_p)\\(b_1,B_1),\dots,(b_q,B_q)\end{matrix};z\right]=\sum_{n=0}^{\infty}\frac{\prod_{i=1}^{p}\Gamma(a_i+A_i n)}{\prod_{j=1}^{q}\Gamma(b_j+B_j n)}\frac{z^n}{n!}, \tag{7}$$

where $z,a_i,b_j\in C$ and $A_i,B_j\in\Re-\{0\},(i=1,\dots,p;\ j=1,\dots,q)$.

Wright proved several theorems on the asymptotic expansion of generalized Wright function ${}_p\psi_q(z)$ for all values of the argument z under the condition

$$\sum_{j=1}^{q} B_j - \sum_{i=1}^{p} A_i > -1 \tag{8}$$

The Srivastava polynomials(also known as general class of polynomials) $S_n^m[x]$ defined by Srivastava [11, 12] as follows:

$$S_n^m[x] = \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} \quad A_{n,k}\ x^k, \qquad (n = 0,1,2,...), \tag{9}$$

where, m is an arbitrary positive integer and the coefficients $A_{n,k}\,(n,k \geq 0)$ are arbitrary constants, real or complex.

2. Generalized fractional calculus operators

Let $\alpha,\alpha',\beta,\beta',\gamma \in C, x > 0$ and $\Re(\gamma) > 0,$ then the generalized fractional integral operators involving Appell's function $F_3(.)$ are introduced by Saigo and Maeda, as follows (see [7],p-393,eq(4.12) and (4.13)):

$$\left(I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma} f\right)(x) = \frac{x^{-\alpha}}{\Gamma(\gamma)} \int_0^x (x-t)^{\gamma-1} t^{-\alpha'} F_3\left(\alpha,\alpha',\beta,\beta';\gamma;1-\frac{t}{x},1-\frac{x}{t}\right) f(t)\,dt, \tag{10}$$

and

$$\left(I_{-}^{\alpha,\alpha',\beta,\beta',\gamma} f\right)(x) = \frac{x^{-\alpha'}}{\Gamma(\gamma)} \int_x^\infty (t-x)^{\gamma-1} t^{-\alpha} F_3\left(\alpha,\alpha',\beta,\beta';\gamma;1-\frac{x}{t},1-\frac{t}{x}\right) f(t)\,dt, \tag{11}$$

The generalized fractional differentiation operators [7] involving the Appell's function $F_3(.)$ as kernel are defined by

$$\left(D_{0+}^{\alpha,\alpha',\beta,\beta',\gamma} f\right)(x) = \left(I_{0+}^{-\alpha',-\alpha,-\beta',-\beta,-\gamma} f\right)(x) \tag{12}$$

$$= \left(\frac{d}{dx}\right)^k \left(I_{0+}^{-\alpha',-\alpha,-\beta'+k,-\beta,-\gamma+k} f\right)(x), \left(\Re(\gamma) > 0; k = \left[\Re(\gamma)+1\right]\right) \tag{13}$$

and

$$\left(D_{-}^{\alpha,\alpha',\beta,\beta',\gamma} f\right)(x) = \left(I_{-}^{-\alpha',-\alpha,-\beta',-\beta,-\gamma} f\right)(x) \tag{14}$$

$$= \left(\frac{-d}{dx}\right)^k \left(I_{-}^{-\alpha',-\alpha,-\beta',-\beta+k,-\gamma+k} f\right)(x), \left(\Re(\gamma) > 0; k = \left[\Re(\gamma)+1\right]\right) \tag{15}$$

Let $\alpha, \beta, \gamma \in C$ and $x > 0$, then the Saigo fractional integral and differential operators [6] associated with Gauss hypergeometric function are defined for $\Re(\alpha) > 0$, as follows:

$$\left(I_{0+}^{\alpha,\beta,\gamma} f\right)(x) = \frac{x^{-\alpha-\beta}}{\Gamma(\gamma)} \int_0^x (x-t)^{\alpha-1} \, {}_2F_1\left(\alpha+\beta, -\gamma; \alpha; 1-\frac{t}{x}\right) f(t)\, dt, \tag{16}$$

$$\left(I_{-}^{\alpha,\beta,\gamma} f\right)(x) = \frac{1}{\Gamma(\gamma)} \int_x^\infty (t-x)^{\alpha-1} \, t^{-\alpha-\beta} \, {}_2F_1\left(\alpha+\beta, -\gamma; \alpha; 1-\frac{x}{t}\right) f(t)\, dt, \tag{17}$$

and

$$\left(D_{0+}^{\alpha,\beta,\gamma} f\right)(x) = \left(I_{0+}^{-\alpha,-\beta,\alpha+\gamma} f\right)(x) = \left(\frac{d}{dx}\right)^k \left(I_{0+}^{-\alpha+k,-\beta-k,\alpha+\gamma-k} f\right)(x), \tag{18}$$

$$\left(D_{-}^{\alpha,\beta,\gamma} f\right)(x) = \left(I_{-}^{-\alpha,-\beta,\alpha+\gamma} f\right)(x) = \left(\frac{-d}{dx}\right)^k \left(I_{-}^{-\alpha+k,-\beta-k,\alpha+\gamma} f\right)(x), \tag{19}$$

where, $k = [\Re(\alpha)] + 1.$

Remark 2: If we take $\alpha = \alpha + \beta, \alpha' = \beta' = 0, \beta = -\gamma$ and $\gamma = \alpha$, operators given in (10)-(15) reduce to the Saigo fractional calculus operators as given by (16)-(19) and if we set $\beta = -\alpha$, then operators (16)-(19) reduce to Riemann-Liouville fractional calculus operators[1].

Further, the following image formulas for a power function, under operators (10) and (1) are given by

$$\left(I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma} t^{\rho-1}\right)(x) = x^{\rho-\alpha-\alpha'+\gamma-1} \, \Gamma\left[\begin{matrix} \rho, \rho+\gamma-\alpha-\alpha'-\beta, \rho+\beta'-\alpha' \\ \rho+\gamma-\alpha-\alpha', \rho+\gamma-\alpha'-\beta, \rho+\beta' \end{matrix}\right], \tag{20}$$

where, $\Re(\gamma) > 0$, $\Re(\rho) > max[0, \Re(\alpha+\alpha'+\beta-\gamma), \Re(\alpha'-\beta')]$,

and

$$\left(I_{-}^{\alpha,\alpha',\beta,\beta',\gamma} t^{\rho-1}\right)(x) = x^{\rho-\alpha-\alpha'+\gamma-1} \, \Gamma\left[\begin{matrix} 1+\alpha+\alpha'-\gamma-\rho, 1+\alpha+\beta'-\gamma-\rho, 1-\beta-\rho \\ 1-\rho, 1+\alpha+\alpha'+\beta'-\gamma-\rho, 1+\alpha-\beta-\rho \end{matrix}\right]. \tag{21}$$

Here, the Symbol $\Gamma\left[\begin{matrix} a,b,c \\ d,e,f \end{matrix}\right]$ will be used to represent the ratio of product of gamma functions as $\dfrac{\Gamma(a)\Gamma(b)\Gamma(c)}{\Gamma(d)\Gamma(e)\Gamma(f)}$.

3. Main Results

3.1 Left-sided and Right-sided Generalized Fractional Integration of Product of Generalized Mittag-Leffler Function and Srivastava Polynomials

In this section, we establish two image formulas for the product of generalized Mittag-Leffler function and Srivastava polynomials involving generalized fractional integral operators also known as Marichev-Saigo-Maeda fractional integral operators (10) and (11), in terms of the generalized Wright function. These formulas are given by the following theorems:

Theorem 1 Let $a,\alpha,\alpha',\beta,\beta',\gamma,\delta,\xi,\rho \in C$, $x,\nu,p,q>0,\ q \le \Re(\nu)+p$ and $\Re(\gamma)>0$ be such that $\Re(\rho)> max\left[0,\Re(\alpha+\alpha'+\beta-\gamma),\Re(\alpha'-\beta')\right]$, then the left-sided generalized fractional integration $I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}$ of the product of generalized Mittag-Leffler function $E_{\nu,\rho,p}^{\delta,\xi,q}(.)$ and Srivastava polynomials $S_n^m[.]$ is given by

$$\{I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\rho-1}S_n^m\left[ct^{\mu}\right]E_{\nu,\rho,p}^{\delta,\xi,q}\left(at^{\nu}\right)\right)\}\left(x\right)=\frac{x^{\rho+\gamma-\alpha-\alpha'-1}\Gamma(\xi)}{\Gamma(\delta)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(cx^{\mu}\right)^{s}$$

$$\times\ {}_5\psi_5\left[\begin{matrix}(1,1),(\delta,q),(\rho+\mu s,\nu),(\rho+\mu s+\gamma-\alpha-\alpha'-\beta,\nu),(\rho+\mu s+\beta'-\alpha',\nu)\\ (\rho,\nu),(\xi,p),(\rho+\mu s+\gamma-\alpha-\alpha',\nu),(\rho+\mu s+\gamma-\alpha'-\beta,\nu),(\rho+\mu s+\beta',\nu)\end{matrix};ax^{\nu}\right] \quad (22)$$

Proof: By using series representation of generalized Mittag-Leffler function (5), Srivastava polynomials (9) and left-sided fractional integration power function formula (20), we have

$$\left\{I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\rho-1}S_n^m\left[ct^{\mu}\right]E_{\nu,\rho,p}^{\delta,\xi,q}\left(at^{\nu}\right)\right)\right\}\left(x\right)$$

$$=\left\{I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\rho-1}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(ct^{\mu}\right)^{s}\sum_{k=0}^{\infty}\frac{(\delta)_{qk}}{\Gamma(\nu k+\rho)(\xi)_{pk}}\left(at^{\nu}\right)^{k}\right)\right\}(x),$$

by interchanging the order of integration and summations, which is valid under the conditions stated with (22), we get

$$=\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\,c^{s}\sum_{k=0}^{\infty}\frac{(\delta)_{qk}\,a^{k}}{\Gamma(\nu k+\rho)(\xi)_{pk}}\left(I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}t^{\rho+\mu s+\nu k-1}\right)(x)$$

$$=\frac{x^{\rho+\gamma-\alpha-\alpha'-1}\Gamma(\xi)}{\Gamma(\delta)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(cx^{\mu}\right)^{s}$$

$$\times\sum_{k=0}^{\infty}\frac{\Gamma(1+k)\Gamma(\delta+qk)\Gamma(\rho+\mu s+\nu k)\Gamma(\rho+\mu s+\gamma-\alpha-\alpha'-\beta+\nu k)}{\Gamma(\nu k+\rho)\Gamma(\xi+pk)\Gamma(\rho+\mu s+\gamma-\alpha-\alpha'+\nu k)\Gamma(\rho+\mu s+\gamma-\alpha'-\beta+\nu k)}$$

$$\times\frac{\Gamma(\rho+\mu s+\beta'-\alpha'+\nu k)}{\Gamma(\rho+\mu s+\beta'+\nu k)}\frac{(ax^{\nu})^k}{k!}. \quad (23)$$

Interpreting the terms of the above equation (23), in view of definition (7), we arrive at the result (22), which completes the proof of theorem 1.

On setting $n=0, A_{0,0}=1$ then $S_0^m[x]\to 1$ in (22), we obtained the following known result given by Gupta et al. $([3], p-560, eq.(3.2))$:

Corollary 1.1 Let $a,\alpha,\alpha',\beta,\beta',\gamma,\delta,\xi,\rho\in C,\ x,\nu,p,q>0, q\le\Re(\nu)+p$ and $\Re(\gamma)>0$ such that $\Re(\rho)>max[0,\Re(\alpha+\alpha'+\beta-\gamma),\Re(\alpha'-\beta')]$, then there holds the formula

$$\{I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\rho-1}E_{\nu,\rho,p}^{\delta,\xi,q}\left(at^{\nu}\right)\right)\}(x)=\frac{x^{\rho+\gamma-\alpha-\alpha'-1}\Gamma(\xi)}{\Gamma(\delta)}$$

$$\times\ {}_4\psi_4\left[\begin{matrix}(\delta,q),(1,1),(\rho+\gamma-\alpha-\alpha'-\beta,\nu),(\rho+\beta'-\alpha',\nu)\\(\xi,p),(\rho+\gamma-\alpha-\alpha',\nu),(\rho+\gamma-\alpha'-\beta,\nu),(\rho+\beta',\nu)\end{matrix};ax^{\nu}\right] \quad (24)$$

on taking $\alpha=\alpha+\beta,\alpha'=\beta'=0,\beta=-\eta$ and $\gamma=\alpha$, Theorem 1 reduces to the following corollary:

Corollary 1.2 *Let* $a,\alpha,\beta,\mu,\rho,\delta,\xi\in C,\ x,\nu,p,q>0, q\le\Re(\nu)+p$ *be such that* $\Re(\alpha)>0$ *and* $\Re(\rho)>max[0,\Re(\beta-\eta)]$, *then there holds the following formula*

$$\{I_{0+}^{\alpha,\beta,\eta}\left(t^{\rho-1}S_n^m\left[ct^{\mu}\right]E_{\nu,\rho,p}^{\delta,\xi,q}\left(at^{\nu}\right)\right)\}(x)=\frac{x^{\rho-\beta-1}\Gamma(\xi)}{\Gamma(\delta)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(cx^{\mu}\right)^s$$

$$\times\ {}_4\psi_4\left[\begin{matrix}(1,1),(\delta,q),(\rho+\mu s,\nu),(\rho+\mu s-\beta+\eta,\nu)\\(\rho,\nu),(\xi,p),(\rho+\mu s-\beta,\nu),(\rho+\mu s+\alpha+\eta,\nu)\end{matrix};ax^{\nu}\right] \quad (25)$$

Remark 3: If we set $n=0, A_{0,0}=1$ then $S_0^m[x]\to 1$ in above corollary (1.2), then we obtain the known result given by Gupta and Parihar $([2], p-140, eq.(2.1))$.

Now, on putting $\beta=-\alpha$ in corollary (2), we arrive at the following result:

Corollary 1.3 Let $a,\alpha,\rho,\delta,\xi\in C,\ x,\nu,p,q>0, q\le\Re(\nu)+p$ be such that $\Re(\alpha)>0$, then there holds the following formula

$$\{I_{0+}^{\alpha}\left(t^{\rho-1}S_n^m\left[ct^{\mu}\right]E_{\nu,\rho,p}^{\delta,\xi,q}\left(at^{\nu}\right)\right)\}\left(x\right)=\frac{x^{\rho+\alpha-1}\Gamma(\xi)}{\Gamma(\delta)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(cx^{\mu}\right)^s$$

$$\times\ {}_3\psi_3[\begin{matrix}(1,1),(\delta,q),(\rho+\mu s,\nu)\\(\rho,\nu),(\xi,p),(\rho+\mu s+\alpha,\nu)\end{matrix};ax^{\nu}] \tag{26}$$

Remark 4: If we take $n=0, A_{0,0}=1$ then $S_0^m\left[x\right]\to 1$ and $\xi=p=q=1$ in equation (26), we obtain the known result given by Saxena and Saigo ([10], p-145, eq(14)).

Remark 5: If we put $n=0, A_{0,0}=1$ then $S_0^m\left[x\right]\to 1$ and $\delta=\xi=p=q=1$ in equation (26), we get the known result given by Samko et al. ([9], table(9.1), formula(23)) (see also [10], p-146, eq.(15)).

Theorem 2 Let $a,\alpha,\alpha',\beta,\beta',\gamma,\delta,\xi,\rho\in C$, $x,\nu,p,q>0,\ q\le\Re(\nu)+p$ and $\Re(\gamma)>0$ be such that $\Re(1-\gamma-\rho)<1+min\left[\Re(-\beta),\Re(\alpha+\alpha'-\gamma),\Re(\alpha+\beta'-\gamma)\right]$, then the right-sided generalized fractional integration $I_{-}^{\alpha,\alpha',\beta,\beta',\gamma}$ of the product of generalized Mittag-Leffler function $E_{\nu,\rho,p}^{\delta,\xi,q}(.)$ and Srivastava polynomials $S_n^m[.]$ is given by

$$\{I_{-}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{-\gamma-\rho}S_n^m\left[ct^{\mu}\right]E_{\nu,\rho,p}^{\delta,\xi,q}\left(at^{-\nu}\right)\right)\}\left(x\right)=\frac{x^{\rho-\alpha-\alpha'}\Gamma(\xi)}{\Gamma(\delta)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(cx^{\mu}\right)^s$$

$$\times\ {}_5\psi_5[\begin{matrix}(1,1),(\delta,q),(\rho-\mu s+\alpha+\alpha',\nu),(\rho-\mu s+\alpha+\beta',\nu),(\rho-\mu s-\beta+\gamma,\nu)\\(\rho,\nu),(\xi,p),(\rho-\mu s+\gamma,\nu),(\rho-\mu s+\alpha+\alpha'+\beta',\nu),(\rho-\mu s+\alpha-\beta+\gamma,\nu)\end{matrix};ax^{-\nu}] \tag{27}$$

Proof: By using series representation of generalized Mittag-Leffler function (5), Srivastava polynomials (9) and right-sided Saigo-Maeda fractional integration power function formula (21), we have

$$\left\{I_{-}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{-\gamma-\rho}S_n^m\left[ct^{\mu}\right]E_{\nu,\rho,p}^{\delta,\xi,q}\left(at^{-\nu}\right)\right)\right\}\left(x\right)$$

$$=\left\{I_{-}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{-\gamma-\rho}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(ct^{\mu}\right)^s\sum_{k=0}^{\infty}\frac{(\delta)_{qk}}{\Gamma\left(\nu k+\rho\right)\left(\xi\right)_{pk}}\left(at^{-\nu}\right)^k\right)\right\}\left(x\right),$$

by interchanging the order of integration and summations, we arrive at the following:

$$=\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\,c^s\sum_{k=0}^{\infty}\frac{(\delta)_{qk}\,a^k}{\Gamma\left(\nu k+\rho\right)\left(\xi\right)_{pk}}\left(I_{-}^{\alpha,\alpha',\beta,\beta',\gamma}t^{-\nu k+\mu s-\rho-\gamma}\right)\left(x\right)$$

$$= \frac{x^{-\rho-\alpha-\alpha'}\Gamma(\xi)}{\Gamma(\delta)} \sum_{s=0}^{[n/m]} \frac{(-n)_{ms}}{s!} A_{n,s} \left(cx^{\mu}\right)^{s}$$

$$\times \sum_{k=0}^{\infty} \frac{\Gamma(\delta+qk)\Gamma(1+k)\Gamma(\rho-\mu s+\alpha+\alpha'+\nu k)\Gamma(\rho-\mu s+\alpha+\beta'+\nu k)}{\Gamma(\nu k+\rho)\Gamma(\xi+pk)\Gamma(\rho-\mu s+\gamma+\nu k)\Gamma(\rho-\mu s+\alpha+\alpha'+\beta'+\nu k)}$$

$$\times \frac{\Gamma(\rho-\mu s-\beta+\gamma+\nu k)}{\Gamma(\rho-\mu s+\alpha-\beta+\gamma+\nu k)} \frac{\left(ax^{-\nu}\right)^{k}}{k!} \tag{28}$$

Now, interpreting the terms of the above equation (28), in view of definition (7), we arrive at the result (27), which completes the proof of theorem 2.

On setting $n = 0, A_{0,0} = 1$ then $S_0^m[x] \to 1$ in equation (27), we obtain the following known result given by Gupta et al. $([3], p-561, eq.(3.7))$.

Corollary 2.1 Let $a,\alpha,\alpha',\beta,\beta',\gamma,\delta,\xi,\rho \in C,\ x,\nu,\ p,q > 0, q \le \Re(\nu)+p$ and $\Re(\gamma) > 0$ such that $\Re(1-\gamma-\rho) < 1+min\left[\Re(-\beta),\Re(\alpha+\alpha'-\gamma),\Re(\alpha+\beta'-\gamma)\right]$, then there holds the formula:

$$\{I_{-}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{-\gamma-\rho}E_{\nu,\rho,p}^{\delta,\xi,q}\left(at^{-\nu}\right)\right)\}(x) = \frac{x^{-\rho-\alpha-\alpha'}\Gamma(\xi)}{\Gamma(\delta)}$$

$$\times {}_5\psi_5\left[\begin{matrix}(1,1),(\delta,q),(\rho+\alpha+\alpha',\nu),(\rho+\alpha+\beta',\nu),(\rho-\beta+\gamma,\nu)\\(\rho,\nu),(\xi,p),(\rho+\gamma,\nu),(\rho+\alpha+\alpha'+\beta',\nu),(\rho+\alpha-\beta+\gamma,\nu)\end{matrix};ax^{-\nu}\right] \tag{29}$$

If we take $\alpha = \alpha+\beta, \alpha' = \beta' = 0, \beta = -\eta$ and $\gamma = \alpha$, Theorem 2 reduces to the following corollary:

Corollary 2.2 Let $a,\alpha,\beta,\eta,\rho,\delta,\xi \in C,\ x,\nu,\ p,q > 0, q \le \Re(\nu)+p$ and $\Re(\alpha) > 0$, such that $\Re(1-\gamma-\rho) < 1+min\left[\Re(-\beta),\Re(-\eta)\right]$, then we have

$$\{I_{-}^{\alpha,\beta,\eta}\left(t^{-\gamma-\rho}S_n^m\left[ct^{\mu}\right]E_{\nu,\rho,p}^{\delta,\xi,q}\left(at^{-\nu}\right)\right)\}(x) = \frac{x^{\rho-\alpha-\beta}\Gamma(\xi)}{\Gamma(\delta)} \sum_{s=0}^{[n/m]} \frac{(-n)_{ms}}{s!} A_{n,s}\left(cx^{\mu}\right)^{s}$$

$$\times {}_4\psi_4\left[\begin{matrix}(1,1),(\delta,q),(\rho-\mu s+\alpha+\beta,\nu),(\rho-\mu s+\alpha+\eta,\nu)\\(\rho,\nu),(\xi,p),(\rho-\mu s+\alpha,\nu),(\rho-\mu s+2\alpha+\beta+\eta,\nu)\end{matrix};ax^{-\nu}\right] \tag{30}$$

Remark 6: If we set $n = 0, A_{0,0} = 1$ then $S_0^m\left[x\right] \to 1$ in equation (30), we obtain the known result given by Gupta and Parihar $([2], p-141, eq.(2.3))$.

Further, if we put $\beta=-\alpha$ in corollary (2.2), we arrive at the following result:

Corollary 2.3 Let $a,\alpha,\rho,\delta,\xi\in C,\ x,\nu,p,q>0,q\le\Re(\nu)+p$ be such that $\Re(\alpha)>0$, then we have

$$\{I_{-}^{\alpha}\left(t^{-\gamma-\rho}S_n^m\left[ct^{\mu}\right]E_{\nu,\rho,p}^{\delta,\xi,q}\left(at^{-\nu}\right)\right)\}(x)=\frac{x^{-\rho}\Gamma(\xi)}{\Gamma(\delta)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(cx^{\mu}\right)^{s}$$

$$\times\ {}_3\psi_3\left[\begin{matrix}(1,1),(\delta,q),(\rho-\mu s,\nu)\\(\rho,\nu),(\xi,p),(\rho-\mu s+\alpha,\nu)\end{matrix};ax^{-\nu}\right] \tag{31}$$

Remark 7: If we take $n=0,A_{0,0}=1$ then $S_0^m\left[x\right]\to 1$ and $\xi=p=q=1$ in equation (31), we obtain the known result given by Saxena and Saigo ([10], p-147, eq(23)).

Remark 8: If we set $n=0,A_{0,0}=1$ then $S_0^m\left[x\right]\to 1$ and $\delta=\xi=p=q=1$ in equation (31), we get the known result (see [10], p-148, eq.(24)).

3.2 Left-sided and right-sided generalized fractional differentiation of product of generalized Mittag-Leffler function and Srivastava polynomials

Now, we shall establish two image formulas for the product of generalized Mittag-Leffler function and Srivastava polynomials involving left-sided and right-sided operators of generalized fractional differentiation operators also called Marichev-Saigo-Maeda fractional differentiation operators(2.3) and (2.5), in terms of the generalized Wright function. These formulas are given by the following theorems:

Theorem 3 Let $a,\alpha,\alpha',\beta,\beta',\gamma,\delta,\xi,\rho\in C$, $x,\nu,p,q>0$, $q\le\Re(\nu)+p$ and $\Re(\gamma)>0$ be such that $\Re(\rho)>max\left[0,\Re(\gamma-\alpha-\alpha'-\beta'),\Re(\beta-\alpha)\right]$, then the left-sided generalized fractional differentiation $D_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}$ of the product of generalized Mittag-Leffler function $E_{\nu,\rho,p}^{\delta,\xi,q}(.)$ and Srivastava polynomials $S_n^m[.]$ is given by

$$\{D_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\rho-1}S_n^m\left[ct^{\mu}\right]E_{\nu,\rho,p}^{\delta,\xi,q}\left(at^{\nu}\right)\right)\}(x)=\frac{x^{\rho-\gamma+\alpha+\alpha'-1}\Gamma(\xi)}{\Gamma(\delta)}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(cx^{\mu}\right)^{s}$$

$$\times\ {}_5\psi_5\left[\begin{matrix}(1,1),(\delta,q),(\rho+\mu s,\nu),(\rho+\mu s-\gamma+\alpha+\alpha'+\beta',\nu),(\rho+\mu s+\alpha-\beta,\nu)\\(\rho,\nu),(\xi,p),(\rho+\mu s-\gamma+\alpha+\alpha',\nu),(\rho+\mu s-\gamma+\alpha+\beta',\nu),(\rho+\mu s-\beta,\nu)\end{matrix};ax^{\nu}\right] \tag{32}$$

Proof: By using (5) and (9), writing the function in the series form and with the help of (12) and (20), L.H.S. of (32) leads to

$$\left\{D_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\rho-1}S_n^m\left[ct^{\mu}\right]E_{\nu,\rho,p}^{\delta,\xi,q}\left(at^{\nu}\right)\right)\right\}(x)$$

$$= \left\{ D_{0+}^{\alpha,\alpha',\beta,\beta',\gamma} \left(t^{\rho-1} \sum_{s=0}^{[n/m]} \frac{(-n)_{ms}}{s!} A_{n,s} \left(ct^{\mu}\right)^{s} \sum_{k=0}^{\infty} \frac{(\delta)_{qk}}{\Gamma(\nu k+\rho)(\xi)_{pk}} \left(at^{\nu}\right)^{k} \right) \right\}(x),$$

by interchanging the order of differentiation and summations, we have

$$= \sum_{s=0}^{[n/m]} \frac{(-n)_{ms}}{s!} A_{n,s}\, c^{s} \sum_{k=0}^{\infty} \frac{(\delta)_{qk}\, a^{k}}{\Gamma(\nu k+\rho)(\xi)_{pk}} \left(D_{0+}^{\alpha,\alpha',\beta,\beta',\gamma} t^{\rho+\mu s+\nu k-1} \right)(x)$$

$$= \sum_{s=0}^{[n/m]} \frac{(-n)_{ms}}{s!} A_{n,s}\, c^{s} \sum_{k=0}^{\infty} \frac{(\delta)_{qk}\, a^{k}}{\Gamma(\nu k+\rho)(\xi)_{pk}} \left(I_{0+}^{-\alpha',-\alpha,-\beta',-\beta,-\gamma} t^{\rho+\mu s+\nu k-1} \right)(x)$$

$$= \frac{x^{\rho-\gamma+\alpha+\alpha'-1}\Gamma(\xi)}{\Gamma(\delta)} \sum_{s=0}^{[n/m]} \frac{(-n)_{ms}}{s!} A_{n,s} \left(cx^{\mu}\right)^{s}$$

$$\times \sum_{k=0}^{\infty} \frac{\Gamma(\delta+qk)\Gamma(1+k)\Gamma(\rho+\mu s+\nu k)\Gamma(\rho+\mu s-\gamma+\alpha+\alpha'+\beta'+\nu k)}{\Gamma(\xi+pk)\Gamma(\rho+\nu k)\Gamma(\rho+\mu s-\gamma+\alpha+\alpha'+\nu k)\Gamma(\rho+\mu s-\gamma+\alpha+\beta'+\nu k)}$$

$$\times \frac{\Gamma(\rho+\mu s+\alpha-\beta+\nu k)}{\Gamma(\rho+\mu s-\beta+\nu k)} \frac{\left(ax^{\nu}\right)^{k}}{k!}, \tag{33}$$

now, by interpreting the terms of the above equation (33), in view of definition (7), we arrive at the result (32), which completes the proof of theorem 3.

On setting $n=0, A_{0,0}=1$ then $S_{0}^{m}[x]\to 1$ in (32), we obtain the following known result given by Gupta et al. $\left([3], p-563, eq.(4.2)\right)$.

Corollary 3.1 Let $a,\alpha,\alpha',\beta,\beta',\gamma,\delta,\xi,\rho \in C,\ x,\nu,p,q>0, q\le \Re(\nu)+p$ and $\Re(\gamma)>0$ be such that $\Re(\rho)>max\left[0,\Re(\gamma-\alpha-\alpha'-\beta'),\Re(\beta-\alpha)\right]$, then there holds the formula :

$$\{D_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\rho-1}E_{\nu,\rho,p}^{\delta,\xi,q}\left(at^{\nu}\right)\right)\}(x) = \frac{x^{\rho-\gamma+\alpha+\alpha'-1}\Gamma(\xi)}{\Gamma(\delta)}$$

$$\times\ {}_4\psi_4\left[\begin{matrix} (1,1),(\delta,q),(\rho-\gamma+\alpha+\alpha'+\beta',\nu),(\rho+\alpha-\beta,\nu) \\ (\xi,p),(\rho-\gamma+\alpha+\alpha',\nu),(\rho-\gamma+\alpha+\beta',\nu),(\rho-\beta,\nu) \end{matrix} ; ax^{\nu} \right] \tag{34}$$

If we put $\alpha=\alpha+\beta, \alpha'=\beta'=0, \beta=-\eta$ and $\gamma=\alpha$, Theorem 3 reduces to the following result:

Corollary 3.2 Let $a,\alpha,\beta,\eta,\rho,\delta,\xi\in C,\ x,\nu,p,q>0, q\le\Re(\nu)+p$ be such that $\Re(\alpha)>0$ and $\Re(\rho)>max\left[0,\Re(\beta-\eta)\right]$, then we have

$$\{D_{0+}^{\alpha,\beta,\eta}\left(t^{\rho-1}S_n^m\left[ct^{\mu}\right]E_{\nu,\rho,p}^{\delta,\xi,q}\left(at^{\nu}\right)\right)\}\left(x\right)=\frac{x^{\rho+\beta-1}\Gamma(\xi)}{\Gamma(\delta)}\sum_{s=0}^{[n/m]}\frac{\left(-n\right)_{ms}}{s!}A_{n,s}\left(cx^{\mu}\right)^s$$

$$\times\ {}_3\psi_3\left[\begin{matrix}(\delta,q),(1,1),(\rho+\mu s+\alpha+\beta+\eta,\nu)\\(\xi,p),(\rho+\mu s+\beta,\nu),(\rho+\mu s+\eta,\nu)\end{matrix};ax^{\nu}\right]\qquad(35)$$

Remark 9: If we take $n=0, A_{0,0}=1$ then $S_0^m\left[x\right]\to1$ in above corollary (23), then we obtain the known result given by Gupta and Parihar $\left([2], p-142, eq.(2.4)\right)$.

Now, if we set $\beta=-\alpha$ in corollary (23), we arrive at the following result:

Corollary 3.3 Let $a,\alpha,\rho,\delta,\xi\in C,\ x,\nu,p,q>0, q\le\Re(\nu)+p$ be such that $\Re(\alpha)>0$, then we have

$$\{D_{0+}^{\alpha}\left(t^{\rho-1}S_n^m\left[ct^{\mu}\right]E_{\nu,\rho,p}^{\delta,\xi,q}\left(at^{\nu}\right)\right)\}\left(x\right)=\frac{x^{\rho-\alpha-1}\Gamma(\xi)}{\Gamma(\delta)}\sum_{s=0}^{[n/m]}\frac{\left(-n\right)_{ms}}{s!}A_{n,s}\left(cx^{\mu}\right)^s$$

$$\times\ {}_2\psi_2\left[\begin{matrix}(1,1),(\delta,q)\\(\xi,p),(\rho+\mu s-\alpha,\nu)\end{matrix};ax^{\nu}\right]\qquad(36)$$

Remark 10: If we put $n=0, A_{0,0}=1$ then $S_0^m\left[x\right]\to1$ and $\xi=p=q=1$ in equation (36), we obtain the known result given by Saxena and Saigo ([10], p-149, eq(29)).

Remark 11: If we set $n=0, A_{0,0}=1$ then $S_0^m\left[x\right]\to1$ and $\delta=\xi=p=q=1$ in equation (36), we get the known result (see [10], p-149, eq.(30)).

Theorem 4 Let $a,\alpha,\alpha',\beta,\beta',\gamma,\delta,\xi,\rho\in C$, $x,\nu,p,q>0$, $q\le\Re(\nu)+p$ and $\Re(\gamma)>0$ be such that $\Re(1-\gamma-\rho)<1+min\left[\Re(-\beta),\Re(\alpha+\alpha'-\gamma),\Re(\alpha+\beta'-\gamma)\right]$, then the right-sided generalized fractional differentiation $D_{-}^{\alpha,\alpha',\beta,\beta',\gamma}$ of the product of generalized Mittag-Leffler function $E_{\nu,\rho,p}^{\delta,\xi,q}(.)$ and Srivastava polynomials $S_n^m[.]$ is given by

$$\{D_{-}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\gamma-\rho}S_n^m\left[ct^{\mu}\right]E_{\nu,\rho,p}^{\delta,\xi,q}\left(at^{-\nu}\right)\right)\}\left(x\right)=\frac{x^{-\rho+\alpha+\alpha'}\Gamma(\xi)}{\Gamma(\delta)}\sum_{s=0}^{[n/m]}\frac{\left(-n\right)_{ms}}{s!}A_{n,s}\left(cx^{\mu}\right)^s$$

$$\times\ {}_5\psi_5\left[\begin{matrix}(1,1),(\delta,q),(\rho-\mu s-\alpha-\alpha',\nu),(\rho-\mu s-\alpha'-\beta,\nu),(\rho-\mu s-\gamma+\beta',\nu)\\(\rho,\nu),(\xi,p),(\rho-\mu s-\gamma,\nu),(\rho-\mu s-\alpha-\alpha'-\beta,\nu),(\rho-\mu s-\gamma-\alpha'+\beta',\nu)\end{matrix};ax^{-\nu}\right]\qquad(37)$$

Proof: By using (5) and (9) writing the function in the series form and with the help of (14) and (21), the left hand side of (37) can be written as

$$\left\{D_{-}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\gamma-\rho}\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\left(ct^{\mu}\right)^{s}\sum_{k=0}^{\infty}\frac{(\delta)_{qk}}{\Gamma(\nu k+\rho)(\xi)_{pk}}\left(at^{-\nu}\right)^{k}\right)\right\}(x),$$

by interchanging the order of differentiation and summations, we have

$$=\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\,c^{s}\sum_{k=0}^{\infty}\frac{(\delta)_{qk}\,a^{k}}{\Gamma(\nu k+\rho)(\xi)_{pk}}\left(D_{-}^{\alpha,\alpha',\beta,\beta',\gamma}t^{\gamma-\rho+\mu s-\nu k}\right)(x)$$

$$=\sum_{s=0}^{[n/m]}\frac{(-n)_{ms}}{s!}A_{n,s}\,c^{s}\sum_{k=0}^{\infty}\frac{(\delta)_{qk}\,a^{k}}{\Gamma(\nu k+\rho)(\xi)_{pk}}\left(I_{-}^{-\alpha',-\alpha,-\beta',-\beta,-\gamma}t^{(1+\gamma-\rho+\mu s-\nu k)-1}\right)(x)$$

$$=\frac{x^{-\rho+\alpha+\alpha'}\Gamma(\xi)}{\Gamma(\delta)}\sum_{s=0}^{n/m}\frac{(-n)_{ms}}{s!}A_{n,s}\left(cx^{\mu}\right)^{s}$$

$$\times\sum_{k=0}^{\infty}\frac{\Gamma(\delta+qk)\Gamma(1+k)\Gamma(\rho-\mu s-\alpha-\alpha'+\nu k)\Gamma(\rho-\mu s-\alpha'-\beta+\nu k)}{\Gamma(\xi+pk)\Gamma(\rho+\nu k)\Gamma(\rho-\mu s-\gamma+\nu k)\Gamma(\rho-\mu s-\alpha-\alpha'-\beta+\nu k)}$$

$$\times\frac{\Gamma(\rho-\mu s-\gamma+\beta'+\nu k)}{\Gamma(\rho-\mu s-\gamma-\alpha'+\beta'+\nu k)}\frac{\left(ax^{-\nu}\right)^{k}}{k!}. \tag{38}$$

Now, by interpreting the terms of the above equation (38), in view of definition (7), we arrive at the result (37), which completes the proof of theorem 4.

On setting $n=0, A_{0,0}=1$ then $S_{0}^{m}[x]\rightarrow 1$ in equation (37), we get the following known result given by Gupta et al. $\left([3], p-564, eq.(4.7)\right)$:

Corollary 4.1 Let $a,\alpha,\alpha',\beta,\beta',\gamma,\delta,\xi,\rho\in C,\ x,\nu,p,q>0, q\le\Re(\nu)+p$ and $\Re(\gamma)>0$ be such that $\Re(1-\gamma-\rho)<1+min\left[\Re(-\beta),\Re(\alpha+\beta'-\gamma),\Re(\alpha+\beta'-\gamma)\right]$, then there holds the following formula*:*

$$\{D_{-}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\gamma-\rho}E_{\nu,\rho,p}^{\delta,\xi,q}\left(at^{-\nu}\right)\right)\}(x)=\frac{x^{-\rho+\alpha+\alpha'}\Gamma(\xi)}{\Gamma(\delta)}$$

$$\times\,{}_{5}\psi_{5}\left[\begin{matrix}(1,1),(\delta,q),(\rho-\alpha-\alpha',\nu),(\rho-\alpha'-\beta,\nu),(\rho-\gamma+\beta',\nu)\\(\rho,\nu),(\xi,p),(\rho-\gamma,\nu),(\rho-\alpha-\alpha'-\beta,\nu),(\rho-\gamma-\alpha'+\beta',\nu)\end{matrix};ax^{-\nu}\right] \tag{39}$$

If we take $\alpha = \alpha + \beta, \alpha' = \beta' = 0, \beta = -\eta$ and $\gamma = \alpha$, Theorem 4 reduces to the following corollary:

Corollary 4.2 Let $a, \alpha, \beta, \eta, \delta, \xi, \rho \in C,\ x, \nu, p, q > 0, q \le \Re(\nu) + p$ and $\Re(\alpha) > 0$, be such that $\Re(1-\eta-\rho) < 1 + min\left[\Re(-\beta), \Re(-\eta)\right]$, then we have

$$\{D_{-}^{\alpha,\beta,\eta}\left(t^{\gamma-\rho} S_n^m\left[ct^{\mu}\right] E_{\nu,\rho,p}^{\delta,\xi,q}\left(at^{-\nu}\right)\right)\}(x) = \frac{x^{-\rho+\alpha+\beta}\Gamma(\xi)}{\Gamma(\delta)} \sum_{s=0}^{[n/m]} \frac{(-n)_{ms}}{s!} A_{n,s}\left(cx^{\mu}\right)^s$$

$$\times\ {}_4\psi_4\left[\begin{matrix}(1,1),(\delta,q),(\rho-\mu s-\alpha-\beta,\nu),(\rho-\mu s+\eta,\nu)\\(\rho,\nu),(\xi,p),(\rho-\mu s-\alpha,\nu),(\rho-\mu s-\alpha-\beta+\eta,\nu)\end{matrix}; ax^{-\nu}\right] \qquad (40)$$

Remark 12: If we set $n = 0, A_{0,0} = 1$ then $S_0^m\left[x\right] \to 1$ in corollary (4.2), we get the known result given by Gupta and Parihar $\left([2], p-143, eq.(2.5)\right)$.

Now, if we take $\beta = -\alpha$ in corollary (4.2), we arrive at the following result:

Corollary 4.3 Let $a, \alpha, \rho, \xi, \delta \in C,\ x, \nu, p, q > 0, q \le \Re(\nu) + p$, such that $\Re(\alpha) > 0$, then there holds the following formula

$$\{D_{-}^{\alpha}\left(t^{\gamma-\rho} S_n^m\left[ct^{\mu}\right] E_{\nu,\rho,p}^{\delta,\xi,q}\left(at^{-\nu}\right)\right)\}(x) = \frac{x^{-\rho}\Gamma(\xi)}{\Gamma(\delta)} \sum_{s=0}^{[n/m]} \frac{(-n)_{ms}}{s!} A_{n,s}\left(cx^{\mu}\right)^s$$

$$\times\ {}_3\psi_3\left[\begin{matrix}(1,1),(\delta,q),(\rho-\mu s,\nu)\\(\rho,\nu),(\xi,p),(\rho-\mu s-\alpha,\nu)\end{matrix}; ax^{-\nu}\right] \qquad (41)$$

Remark 13: If put $n = 0, A_{0,0} = 1$ then $S_0^m\left[x\right] \to 1$ and $\xi = p = q = 1$ in above equation (41), we obtain the known result given by Saxena and Saigo ([10], p-150, eq(35)).

Remark 14: If take $n = 0, A_{0,0} = 1$ then $S_0^m\left[x\right] \to 1$ and $\delta = \xi = p = q = 1$ in equation (41), we get the known result (see [10], p-151, eq.(36)).

4. Conclusion

In the present paper, we have given four theorems of generalized fractional calculus operators given by Marichev-Saigo-Maeda associated with the product of generalized Mittag-leffler function and Srivasatava polynomials in terms of generalized Wright function. On account of the general nature of the generalized Mittag-Leffler function and generalized Wright function, a number of known results can easily be found as special cases of our main results.

Acknowledgements

The authors are thankful to the Referee for valuable comments and suggestions.

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