Journal of Rajasthan Academy of Physical Sciences
ISSN : 0972-6306; URL : http://raops.org.in
Vol.18, No.1&2, January-June, 2019, 45-56

ON THE CONHARMONIC CURVATURE TENSOR OF A N(K)-CONTACT METRIC MANIFOLD

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Abstract: The object of present paper is to classify N(k)-contact metric manifolds satisfying certain curvature conditions[15] on the conharmonic curvature tensor. Here, we consider quasiconharmonically flat, conharmonically semisymmetric, and φ-conharmonically semisymmetric N(k)-contact metric manifolds. Beside these, we also study Einstein semisymmetric conharmonically flat N(k)-contact metric manifold.

2010 Mathematics Subject Classification: Primary 53C15 ; Secondary 53C25, 53C40.

Keywords: N(k)-contact metric manifold, Conharmonic Curvature tensor, quasiconharmonically flat, conharmonically semisymmetric, Einstein semisymmetric, η-Einstein manifold.

1. Introduction

In a Riemannian manifold M of dimension $n \geq 3$, the conharmonic curvature tensor H is defined by [11]

$$H(X,Y)Z = R(X,Y)Z - \frac{1}{n-2}\{S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY\},$$

for X,Y,Z ∈ TM, where R is the curvature tensor and Q is the Ricci operator.

In a (2n+1) dimensional almost contact metric manifold, the conharmonic curvature tensor H is defined by

$$H(X,Y)Z = R(X,Y)Z - \frac{1}{2n-1}\{S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY\}, \qquad (1)$$

A manifold whose conharmonic curvature tensor vanishes at every point of the manifold is called conharmonically flat manifold. Hence, this tensor represents the deviation of the manifold from conharmonic flatness. There are many physical application of the tensor H. For example, in [1], Abdussattar showed that sufficient condition for a spacetime to be conharmonic to a flat spacetime is that the tensor H vanishes identically. Also, he described the gravitational field due to a distribution of pure radiation in presence of disordered radiation by means of spherically symmetric conharmonically flat spacetime.

Conharmonic curvature tensor have been studied by Siddiqui and Ahsan [16], $\ddot{O}zg\ddot{u}r$[14], and Ishi[11]. Let (M, g) be a Riemannian manifold and let ∇ be the Levi-Civita connection of (M, g). A Riemannian manifold is called locally symmetric if $\nabla R = 0$ [9], where R is the Riemannian curvature tensor of (M, g). A Riemannian manifold M is called semisymmetric if R.R=0 holds, where R is the curvature tensor of the manifold. A fundamental study on Riemannian semisymmetric manifolds was made by Szabo [17], Boeckx et al. [8] and Kowalski [13]. A Riemannian manifold M is said to be Ricci semisymmetric if on M we have R.S=0, where S is the Ricci tensor.

In [4] Blair et al. studied N(k)-contact metric manifold satisfying the curvature conditions $\tilde{Z}.\tilde{Z}$=0, $\tilde{Z}$.R=0 and R.$\tilde{Z}$=0, where $\tilde{Z}$ is the concircular curvature tensor. In [10] Dwivedi and Kim studied quasiconharmonically flat, ξ-conharmonically flat, φ-conharmonically flat k-contact metric manifold. Ghosh et al. [12] studied conharmonically semisymmetric, ξ-conharmonically flat and φ-conharmonically flat N(k)-contact metric manifold.

Motivated by the above studies [4], [10], [12] and the studies of Quasi-conformal curvature tensor on N(k)-contact metric manifolds in [19], we study conharmonic curvature tensor in N(k)-contact metric manifold.

The present paper is organised as follows. In section 2, we give some preliminary results that will we needed thereafter. In section 3, we discuss quasiconharmonically flat N(k)-contact metric manifold and it is shown that the scalar curvature vanishes, and consequently the manifold is η-Einstein. Section 4, we consider conharmonically semisymmetric N(k)-contact metric manifold and prove that the manifold is η-Einstein. Section 5 deals with the study of φ-conharmonically semisymmetric N(k)-contact metric manifold. Finally, the last section is devoted to study Einstein semisymmetric conharmonically flat N(k)-contact metric manifold.

2. Preliminaries

A (2n+1) dimensional differentiable manifold M is said to admit an almost contact structure if it admits a tensor fields φ of type (1,1), a vector field ξ and a 1-form η satisfying [3], [5]

$$\varphi^2 = -I + \eta \otimes \xi,\ \eta(\xi) = 1,\ \varphi\xi = 0,\ \eta o \varphi = 0. \tag{2}$$

An almost contact structure is said to be normal if the induced almost contact structure J on the product manifold M×R defined by

$$J\left(X, \lambda \frac{d}{dt}\right) = \left(\varphi X - \lambda\xi, \eta \otimes \frac{d}{dt}\right)$$

is integrable, where X is tangent to M, t is the coordinate of R and λ is a smooth function on M×R. The condition of almost contact metric structure being normal is equivalent to vanishing of the torsion tensor [φ, φ]+2d $\eta \otimes \xi$, where [φ, φ] is the Nijenhuis tensor of φ. Let g be the compatible Riemannian metric with almost contact structure (φ, ξ, η), that is

$$g(\varphi X,\varphi Y) = g(X,Y) - \eta(X)\,\eta(Y),\ g(X, \xi) = \eta(X) \tag{3}$$

for all vector fields X,Y ∈ TM.

A manifold M together with this almost contact metric structure is said to be almost contact metric manifold denoted by M(φ, ξ, η, g). An almost contact metric structure reduces to a contact metric structure if $g(X, \varphi Y) = d\eta(X, Y)$. Moreover, if ∇ denotes the Riemannian connection of g, then the following relation holds,

$$\nabla_X \xi = -\varphi X - \varphi h X. \quad (4)$$

A contact metric manifold is said to be η-Einstein if

$$S(X,Y) = \alpha\, g(X,Y) + \beta\, \eta(X)\eta(Y),$$

where α and β are smooth functions.

If $\beta = 0$, then the manifold M is an Einstein manifold.

Blair, et al. [6] is introduced the (k,μ)-nullity distribution of a contact metric manifold M that is defined by

$N(k,\mu)$: p$\rightarrow N_p(k,\mu)$

$$N_p(k,\mu) = \{W \in T_pM : R(X,Y)W = (kI + \mu h)(g(Y,W)X - g(X,W)Y)\}$$

for all X,Y ∈ TM, where $(k,\mu) \in R^2$. A contact metric manifold M with $\xi \in N(k,\mu)$ is called a (k,μ) contact metric manifold. If μ = 0, the (k,μ)-nullity distribution reduces to k-nullity distribution [18].

The k-nullity distribution N(k) of a Riemannian manifold is defined by [18]

$$N(k): p \rightarrow N_p(k) = \{W \in T_p(M): R(X,Y)W = k(g(Y,W)X - g(X,W)Y)\},$$

k being a constant.

If the characteristic vector field ξ ∈ $N(k)$,then we call a contact metric manifold as N(k)-contact metric manifold [4]. If $k = 1$, then the manifold is Sasakian and if $k = 0$,then the manifold is isometric to the product $E^{n+1}(0) \times S^n(4)$ for n > 1 and flat for n = 1 [7].

Given a non-Sasakian (k, μ)-contact manifold M, Boeckx [8] introduced an invariant

$$I_M = \frac{1 - \mu/2}{\sqrt{1-k}}$$

and show that for two non-Sasakian (k, μ)- manifolds M_1 and M_2, we have $I_{M_1} = I_{M_2}$ if and only if upto D-homothetic deformation, then two manifolds are locally isometric as contact metric manifolds.

Thus, we see that from all non-Sasakian (k, μ) manifolds of dimension (2n+1) and for every possible value of the invariant I_M, one (k, μ)-manifold M can be obtained. If $I_M > -1$ such examples may be found from the standard contact metric structure on the tangent sphere bundle of a manifold of constant curvature C where we have $I_M = \frac{(1+c)}{|1-c|}$. Boeckx also gives a Lie-algebra construction for any odd dimension and value of $I_M \leq -1$.

Using this invariant, Blair, Kim and Tripathi [4] constructed an example of (2n+1) dimensional $N(1-\frac{1}{n})$-contact metric manifold, n > 1. The example is given as follows:

Since the Boeckx invariant for $(1-\frac{1}{n}, 0)$-manifold is $\sqrt{n} > -1$, we consider the tangent sphere bundle of an (n+1)-dimensional manifold of constant curvature C so chosen that the resulting D-homothetic deformation will be a $(1-\frac{1}{n}, 0)$-manifold. That is, for $k = c(2-c)$ and $\mu = -2c$, we solve

$$1-\frac{1}{n} = \frac{k+a^2-1}{a^2}, \qquad 0 = \frac{\mu+2a-2}{a}$$

for a and c.

The result is

$$c = \frac{\sqrt{n} \pm 1}{n-1}, a = 1 + c$$

for taking c and a to these values we obtain $N(1-\frac{1}{n})$-contact metric manifold.

In an N(k)-contact metric manifold, the following relation holds :

$$h^2 = (k-1)\varphi^2, \text{where } h = \frac{1}{2}£_\xi\, \varphi \tag{5}$$

$$R(X,Y)\xi = k[\eta(Y)X - \eta(X)Y], \tag{6}$$

$$S(X,Y) = 2(n-1)g(X,Y) + 2(n-1)g(hX,Y) +$$
$$[2nk - 2(n-1)]\eta(X)\eta(Y)], n \geq 1 \tag{7}$$

$$QX = 2(n-1)X + 2(n-1)hX + [2(1-n) + 2nk]\eta(X)\xi, n \geq 1. \tag{8}$$

$$S(X,\xi) = 2nk\eta(X), \tag{9}$$

$$r = 2n(2n-2+k), \tag{10}$$

$$(\nabla_X\eta)(Y) = g(X + hX, \varphi Y), \tag{11}$$

In a (2n+1) dimensional almost contact metric manifold, if $\{e_1, e_2 \dots\dots\dots. e_{2n}, \xi\}$ is a local orthonormal basis of tangent space of the manifold, then $\{\varphi e_1, \varphi e_2 \dots\dots\dots. \varphi e_{2n}, \xi\}$ is a local orthonormal basis. It is easy to verify that

$$\sum_{i=1}^{2n} g(e_i, e_i) = \sum_{i=1}^{2n} g(\varphi e_i, \varphi e_i) = 2n, \tag{12}$$

$$\sum_{i=1}^{2n} S(e_i, e_i) = \sum_{i=1}^{2n} S(\varphi e_i, \varphi e_i) = r - 2nk, \tag{13}$$

$$\sum_{i=1}^{2n} g(e_i, Z)\, S(Y, e_i) = \sum_{i=1}^{2n} g(\varphi e_i, Z)\, S(Y, \varphi e_i) = S(Y,Z) - 2nk\eta(Y)\eta(Z), \tag{14}$$

$$\sum_{i=1}^{2n} g(R(e_i,Y)Z,e_i) = \sum_{i=1}^{2n} g(R(\varphi e_i,Y)Z,\varphi e_i)$$

$$=S(Y,Z) - kg(\varphi Y,\varphi Z) \tag{15}$$

for all Y,Z ∈ TM.

Here we state a lemma due to Baikoussis and Koufogiorogos [2] which will be used in this paper.

Lemma 2.1: Let M^{2n+1} be an η-Einstein manifold of dimension (2n+1) $(n \geq 1)$. If ξ belongs to the k-nullity distribution, then $k = 1$ and the structure is Sasakian.

3. Quasiconharmonically flat N(k)-contact metric manifold

Definition 3.1: An almost contact metric manifold M is said to be quasiconharmonically flat[10] if

$$g(H(X,Y)Z,\varphi W) = 0, \tag{16}$$

X,Y,Z,W ∈ TM.

From (1) we get

$$g(H(X,Y)Z,\varphi W) = g(R(X,Y)Z,\varphi W) - \frac{1}{2n-1}\{S(Y,Z)g(X,\varphi W) - S(X,Z)g(Y,\varphi W) + g(Y,Z)S(X,\varphi W) - g(X,Z)S(Y,\varphi W)\} \tag{17}$$

for X,Y,Z,W ∈ TM.

For a local orthonormal basis $\{e_1, e_2 \dots\dots\dots . e_{2n}, \xi\}$of vector fields in M, putting $X = \varphi e_i$ and $W = e_i$ in (17) we get

$$\sum_{i=1}^{2n} g(H(\varphi e_i,Y)Z,\varphi e_i) = \sum_{i=1}^{2n} g(R(\varphi e_i,Y)Z,\varphi e_i) - \frac{1}{2n-1} - \sum_{i=1}^{2n}\{S(Y,Z)g(\varphi e_i,\varphi e_i) - S(\varphi e_i,Z)g(Y,\varphi e_i) + g(Y,Z)S(\varphi e_i,\varphi e_i) - g(\varphi e_i,Z)S(Y,\varphi e_i)\} \tag{18}$$

for Y,Z ∈ TM.

Using the equation (12), (13), (14) and (15) in (18), yields

$$\sum_{i=1}^{2n} g(H(\varphi e_i,Y)Z,\varphi e_i) = S(Y,Z) - kg(\varphi Y,\varphi Z) - \frac{1}{2n-1}\{2(n-1)S(Y,Z) + (r-2nk)g(Y,Z) + 4nk\eta(Y)\eta(Z)\} \tag{19}$$

Using (3) in (19) we get

$$\sum_{i=1}^{2n} g(H(\varphi e_i, Y)Z, \varphi e_i) = \frac{1}{2n-1}\{S(Y,Z) - (r-k)g(Y,Z) - (2n+1)k\,\eta(Y)\eta(Z)\} \quad (20)$$

for Y,Z ∈ TM.

In particular if M is quasiconharmonically flat then (20) reduces to

$$S(Y,Z) = (r-k)g(Y,Z) + (2n+1)k\,\eta(Y)\eta(Z) \quad (21)$$

for Y,Z ∈ TM.

Putting $Z = \xi$ in (21) and using (3), (9) and $\eta(\xi) = 1$, we get

$$r = 0 \quad (22)$$

Hence, we can state the following theorem.

Theorem 3.1: If a (2n+1) dimensional N(k)-contact metric manifold is quasiconharmonically flat then scalar curvature r vanishes.

Using (22) in (21), we have

$$S(Y,Z) = -kg(Y,Z) + (2n+1)k\,\eta(Y)\eta(Z) \quad (23)$$

From relation (23), we conclude that the manifold is an η-Einstein manifold.

Hence, we can state the following theorem.

Theorem 3.2 : A (2n+1) dimensional quasiconharmonically flat N(k)-contact metric manifold is an η-Einstein manifold.

Hence in view of Lemma (2.1) we state the following.

Corollary 3.1 : Let M be a (2n+1) dimensional quasiconharmonically flat N(k)-contact metric manifold, then $k = 1$ and the structure is Sasakian.

Definition 3.2 : In a (2n+1) dimensional N(k)-contact metric manifold if a Ricci tensor S satisfies $\nabla_X\big(S(\varphi Y, \varphi Z)\big) = 0$, then the Ricci tensor is said to be η-parallel.

Replacing Y and Z by φY and φZ in (23) and using (2) we obtain

$$S(\varphi Y, \varphi Z) = -kg(\varphi Y, \varphi Z)$$

Above relation yields

$$(\nabla_X S)(\varphi Y, \varphi Z) = 0$$

Since k is constant. Therefore, we have the following corollary.

Corollary 3.2 : A (2n+1) dimensional quasiconharmonically flat N(k)-contact metric manifold satisfies η-parallel Ricci tensor.

4. Conharmonically Semisymmetric N(k)-Contact Metric Manifold

Let us consider a conharmonically semisymmetric N(k)-contact metric manifold. Then, the condition

$$R(X,Y).H = 0 \tag{24}$$

holds for every vector fields X,Y. From the above equation, we have

$$0 = (R(X,Y).H)(U,V)W = R(X,Y)H(U,V)W - H(R(X,Y)U,V)W - H(U,R(X,Y)V)W - H(U,V)R(X,Y)W. \tag{25}$$

Put $X = \xi$, the above equation gives

$$0 = R(\xi,Y)H(U,V)W - H(R(\xi,Y)U,V)W - H(U,R(\xi,Y)V)W - H(U,V)R(\xi,Y)W. \tag{26}$$

In view of (6), the equation (26) can be written as

$$0 = k[H(U,V,W,Y)\xi - \eta(H(U,V)W)Y + \eta(U)H(Y,V)W - g(Y,W)H(U,V)\xi - g(Y,U)H(\xi,V)W + \eta(V)H(U,Y)W - g(Y,V)H(U,\xi)W + \eta(W)H(U,V)Y], \tag{27}$$

where

$$H(U,V,W,Y) = g(H(U,V)W,Y).$$

Taking inner product of (27) with ξ, we get

$$k[H(U,V,W,Y) - \eta(H(U,V)W)\eta(Y) + \eta(U)\eta(H(Y,V)W) - g(Y,U)\eta(H(\xi,V)W) + \eta(V)\eta(H(U,Y)W) - g(Y,V)\eta(H(U,\xi)W) + \eta(W)\eta(H(U,V)Y) - g(Y,W)\eta(H(U,V)\xi) = 0 \tag{28}$$

Putting $Y = U$ in (28), yields

$$k[H(U,V,W,U) - g(U,U)\eta(H(\xi,V)W) - g(U,V)\eta(H(U,\xi)W) + \eta(W)\eta(H(U,V)U)] = 0. \tag{29}$$

Let $\{e_1, e_2 \ldots\ldots\ldots e_{2n}, \xi\}$ be a local orthonormal basis of tangent space of the manifold. Putting $U = e_i$ in (29) and summing up from 1 to 2n, then by virtue of (1), (6), (9) and (12) to (15), the above equation reduces to

$$S(V,W) = \left\{\frac{r-2nk}{2n}\right\} g(V,W) + \left\{\frac{(2n+1)2nk-r}{2n}\right\} \eta(V)\eta(W) \tag{30}$$

Hence, we can state the following theorem.

Theorem 4.1: If M is a (2n+1) dimensional conharmonically semisymmetric N(k)-contact metric manifold, then the manifold is η-Einstein manifold.

5. φ-Conharmonically Semisymmetric N(k)-Contact Metric Manifold

Definition 5.1: A N(k)-contact metric manifold is said to be φ-conharmonically semisymmetric if

$(H(X,Y).\varphi)W = 0$

for all smooth vector fields X,Y,W.

In this section we deal with φ- conharmonically semisymmetric N(k)-contact metric manifolds. Suppose

$(H(X,Y).\varphi)W = 0$

then,

$$H(X,Y)\varphi W - \varphi(H(X,Y)W = 0 \tag{31}$$

Using (1) and (6) in (31) yields

$$0 = k[g(Y,\varphi W)X - g(Y,W)\varphi X - g(X,\varphi W)Y + g(X,W)\varphi Y] - \frac{1}{2n-1}[S(Y,\varphi W)X - S(Y,W)\varphi X - S(X,\varphi W)Y + S(X,W)\varphi Y + g(Y,\varphi W)QX - g(Y,W)\varphi(QX) - g(X,\varphi W)QY + g(X,W)\varphi(QY)] \tag{32}$$

Putting $Y = W = \xi$ in the above equation and using (2), (9), (10) yields

$$k(\varphi X) + \varphi(QX) = 0 \tag{33}$$

using (8) in (33), we get

$$[2(n-1) + k]\varphi(X) + 2(n-1)\varphi(hX) = 0 \tag{34}$$

Taking inner product with Y in (34) and using (7) yields

$$S(\varphi X, Y) = [4(n-1) + k]g(\varphi X, Y) \tag{35}$$

Replacing Y by φY in the above equation we have

$$S(\varphi X, \varphi Y) = [4(n-1) + k]g(\varphi X, \varphi Y) \tag{36}$$

Put $X = Y = e_i$ in (36) and taking summation over i=1 to 2n+1 we get by using (11), (12) and (13) that

$$r = 0 \tag{37}$$

In view of the above discussion we have the following.

Theorem 5.1: If a (2n+1) dimensional $(n \geq 1)$ N(k)-contact metric manifold M is φ-conharmonically semisymmetric, then the scalar curvature r vanishes.

6. Einstein Semisymmetric Conharmonically Flat N(k)-Contact Metric Manifold

Definition 6.1: [19] The Einstein Tensor denoted by E, is defined by

$$E(X,Y) = S(X,Y) - \frac{r}{2}g(X,Y), \tag{38}$$

where S is Ricci tensor and r is the scalar curvature.

Definition 6.2: A (2n+1) dimensional conharmonically flat N(k)-contact metric manifold is called Einstein semisymmetric if

$$R(X,Y).E(Z,W) = 0 \tag{39}$$

for any vector fields X,Y,Z and W.

We consider on N(k)-contact metric manifold which is conharmonically flat. Then from (1), we get

$$R(X,Y)Z = \frac{1}{2n-1}\{S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY\} \tag{40}$$

Taking inner product on both sides of above equation with respect to W, we get

$$R(X,Y,Z,W) = \frac{1}{2n-1}\{S(Y,Z)g(X,W) - S(X,Z)g(Y,W) + g(Y,Z)S(X,W) - g(X,Z)S(Y,W)\} \tag{41}$$

where $R(X,Y,Z,W) = g(R(X,Y)Z,W)$ and $S(X,Y) = g(QX,Y)$.

Putting $X = W = \xi$ in (41) and using (6), (9) yields

$$S(Y,Z) = -kg(Y,Z) + (2n+1)k\,\eta(Y)\eta(Z) \tag{42}$$

$$S(Y,Z) = Ag(Y,Z) + B\,\eta(Y)\eta(Z) \tag{43}$$

where $A = -k$ and $B = (2n+1)k$

using $g(QX,Y) = S(X,Y)$ in (42), we get

$$QX = AX + B\,\eta(X)\xi \tag{44}$$

Substituting (42) and (44) in (40), we have

$$R(X,Y)Z = M\{g(Y,Z)X - g(X,Z)Y\} + N\{g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi + \eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y\}, \tag{45}$$

where $M = \frac{-2k}{2n-1}$, $N = \frac{(2n+1)k}{2n-1}$.

Now, we consider the conharmonically flat N(k)-contact metric manifold which is Einstein semisymmetric, i.e,

$$R.E = 0 \tag{46}$$

The above equation reduces to

$$E(R(X,Y)Z,U) + E(Z,R(X,Y)U) = 0 \tag{47}$$

In view of (38), equation (47) gives

$$S(R(X,Y)Z,U) - \frac{r}{2}g(R(X,Y)Z,U) + S(Z,R(X,Y)U) - \frac{r}{2}g(Z,R(X,Y)U) = 0 \tag{48}$$

Using (43), we have from the above equations

$$\left(A - \frac{r}{2}\right)g(R(X,Y)Z,U) + \left(A - \frac{r}{2}\right)g(Z,R(X,Y)U) + B\,\eta(R(X,Y)Z)\eta(U) + B\,\eta(R(X,Y)U)\eta(Z) = 0 \tag{49}$$

Put $Z = \xi$ in (49), we get

$$\left(A - \frac{r}{2}\right) g(R(X,Y)\xi, U) + \left(A - \frac{r}{2}\right) g(\xi, R(X,Y)U) + B\,\eta(R(X,Y)\xi)\eta(U) + B\,\eta(R(X,Y)U) = 0 \quad (50)$$

Using (45) in (50), we have

$$B\{g(X,U)\eta(Y) - g(Y,U)\eta(X)\} = 0 \quad (51)$$

Putting $Y = \xi$ in (51), we get

$$B\{g(X,U) - \eta(U)\eta(X)\} = 0 \quad (52)$$

Putting $U = QW$ in (51) and using (44), we have

$$B\{S(X,W) - (A+B)\eta(W)\eta(X)\} = 0 \quad (53)$$

This gives, either $B = 0$ or $S(X,W) - (A+B)\eta(W)\eta(X) = 0$

Now, if $B = 0$, then we have r is constant.

Again, if $S(X,W) - (A+B)\eta(W)\eta(X) = 0$, then we have

$$S(X,W) = (A+B)\eta(W)\eta(X) \quad (54)$$

Putting $X = W = e_i$ in (54) and taking summation over i=1 to 2n+1 we get by using (13) that

$r = 2nk$

Thus, we can state the following theorem.

Theorem 6.1: If a (2n+1) dimensional conharmonically flat N(k)-contact metric manifold is Einstein semisymmetric, then the scalar curvature is constant.

Acknowledgements

The authors are thankful to the referees for their valuable suggestions in the improvement of the paper.

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