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COSMOLOGICAL MODEL FOR RADIATION DOMINATED PHASE WITH DARK ENERGY IN C-FIELD COSMOLOGY

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Abstract: A cosmological model in C-field (Creation field) Cosmology with decaying vacuum energy density (Λ) for radiation dominated phase in the frame work of FRW space-time, is investigated. To get the deterministic model, we have assumed $\Lambda \sim \frac{1}{R^2}$ i.e. $\Lambda = \frac{\alpha}{R^2}$, α being a constant (Chen and Wu [7]) and $\dot{C} = 1$, $\frac{dC}{dt} = 1$ as considered by Hoyle and Narlikar[10]. We find that matter density is positive and creation field increases with time. The singularity in the model is explained in context of creation field cosmology. The vacuum energy decreases with time and tends to zero for large values of time. The model represents accelerating universe as decelerating parameter q < 0.

1. Introduction

It is interesting to study cosmological model in C-field cosmology because it solves the problems of singularity, horizon and flatness of universe which were the out standing problems of Big Bang cosmology (Narlikar and Padmanabhan[11]).

As pointed out by Narlikar and Padmanabhan1[11], these problems may be solved by (i) quantum cosmological model (ii) Inflationary cosmological model. But, we do not have a complete quantum theory of gravity. The inflationary model of Guth[9] does not solve the problem of singularity for creation. In other words, the inflationary phase is always preceded by a hot, radiation dominated epoch which is singular. In Creation field cosmology, the negative energy field is introduced and is explained as; If a model successfully explains creation of positive energy matter without violating the conservation of energy then it is necessary to have same degree of freedom which acts as a negative energy mode (Hoyle and Narlikar[10]). All quantum gravitational models which describe creation consistency, use such a negative energy mode (Atkatz and Pagels[1]). Thus, the negative energy field in C-field cosmology is justified.

In early stage of the universe, the radiation was the dominant influence on the expansion of the universe. The roles of matter and radiation changed after the cooling with the expansion and universe entered a matter dominated era. The study of radiation dominated era, helps to understand the origin and evolution of universe. Recent results suggest that we have entered an era dominated by Dark energy (Frieman et al.[8]). A wide range of observations suggest that cosmological term (Λ) is the most favored candidate of dark energy representing energy density of vacuum.

Barrow and Shaw[4] suggested that cosmological term (Λ) corresponds to a very small value of the order 10^{-122} when applied to Friedmann universe . A number of Dark Energy cosmological models have been investigated by several authors viz. Berman[6], Beesham[5], Saha[12], Bali and Singh[2] in different contexts. Recently Bali and Saraf[3] investigated C-field Cosmological model for dust distribution with time dependent cosmological term (Λ) in FRW space-time.

In this paper, we have investigated cosmological model for radiation dominated phase with Dark energy in C-field cosmology in the framework of FRW space-time. To get the deterministic model of universe, we have assumed $\Lambda \sim \frac{1}{R^2}$, R is scale factor as considered by Chen and Wu[7]. The other physical aspects of the model are also discussed.

2. Metric and Field Equations

We consider the FRW space-time in the form

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2\right] \tag{1}$$

where k = 0, –1, 1.

The Einstein field equation by introduction of C-field is modified by Hoyle and Narlikar[10] with time-dependent Λ given by

$$R_i^j - \frac{1}{2}\bar{R}\, g_i^j = -8\pi G[\underset{(m)}{T}{}_i^j + \underset{(c)}{T}{}_i^j] + \Lambda(t)\; g_i^j \tag{2}$$

where energy momentum tensors $\underset{(m)}{T}{}_i^j$ for viscous and $\underset{(c)}{T}{}_i^j$ for Creation field are given by

$$\underset{(m)}{T}{}_i^j = (\rho + p) v_i\, v^j - p\, g_i^j \tag{3}$$

and

$$\underset{(c)}{T}{}_i^j = -f\left(C_i\, C^j - \frac{1}{2} g_i^j C^\alpha C_\alpha\right) \tag{4}$$

where f > 0 is the coupling constant between matter and creation field, $C_i = \frac{dC}{dx^i}$, ρ the matter density, p the isotropic pressure, $v_i v^i = 1, v_\alpha = 0, \alpha = 1,2,3$

The modified Einstein field equation for the metric (1) with variable Λ(t) leads to

$$\frac{3\dot{R}^2}{R^2} + \frac{3k}{R^2} = 8\pi G\left[\rho - \frac{1}{2} f\,\dot{C}^2\right] + \Lambda(t) \tag{5}$$

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = 8\pi G\left[\frac{1}{2} f\dot{C}^2 - p\right] + \Lambda(t) \tag{6}$$

3. Solution of Field Equations

The conservation equation

$$[8\pi G\, T_i^j + \Lambda g_i^j]_{;j} = 0 \tag{7}$$

leads to

$$8\pi\dot{G}\left(\rho - \frac{1}{2} f\dot{C}^2\right) + 8\pi G\,(\dot{\rho} - f\,\dot{C}\,\ddot{C})$$

$$+ 8\pi G\left[\frac{3\rho\dot{R}}{R} - \frac{3\dot{R}}{R} f\dot{C}^2 + 3p\frac{\dot{R}}{R}\right] + \dot{\Lambda} = 0 \tag{8}$$

We also assume that the universe is radiation dominated (i.e. ρ=3p; p being the isotropic pressure, ρ the matter density).

Using G as a constant and ρ=3p, equation (8) leads to

$$8\pi G\left[\dot{\rho} - f\,\dot{C}\,\ddot{C} + 4\rho\frac{\dot{R}}{R} - \frac{3\dot{R}}{R} f\dot{C}^2\right] + \dot{\Lambda} = 0 \tag{9}$$

Following Hoyle and Narlikar[10], the source equation of C- field $C^i_{;i} = \frac{n}{f}$. which yield $\dot{C} = 1$ leads to C=t for large r.

We have taken $\dot{C} = 1$. Equation (5) and (6) leads to

$$\frac{3\dot{R}^2}{R^2} + \frac{3k}{R^2} = 8\pi G\left[\rho - \frac{1}{2}f\right] + \Lambda(t) \tag{10}$$

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = 8\pi G\left[\frac{1}{2}f - p\right] + \Lambda(t) \tag{11}$$

Using the condition ρ=3p in equation (10) and (11), we have

$$8\pi G\rho = \frac{3\dot{R}^2}{R^2} + \frac{3k}{R^2} + 4\pi G f - \Lambda \tag{12}$$

$$-\frac{8\pi G\rho}{3} = \frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} - 4\pi G f - \Lambda \tag{13}$$

Using (12) and (13), we have

$$\frac{2\ddot{R}}{R} + \frac{2\dot{R}^2}{R^2} = \frac{8\pi G f}{3} + \frac{4\Lambda}{3} - \frac{2k}{R^2} \tag{14}$$

Following Chen and Wu[7], we assume that $\Lambda = \frac{\alpha}{R^2}$ where R is scale factor and α is a constant.

Using $\Lambda = \frac{\alpha}{R^2}$ in equation (14), we have

$$2\ddot{R} + \frac{2\dot{R}^2}{R} = \frac{8\pi G f}{3}R + \frac{1}{R}\left[\frac{4\alpha}{3} - 2k\right] \tag{15}$$

To get the solution, let $\dot{R} = F(R)$ this leads to $\ddot{R} = FF'$ where $F' = \frac{dF}{dR}$

Thus equation (15) leads to

$$\frac{dF^2}{dR} + \frac{2}{R}F^2 = \frac{8\pi G f}{3} + \frac{1}{R}\left[\frac{4\alpha - 6k}{3}\right] \tag{16}$$

Which leads to

$$F^2 = \frac{2\pi G f}{3}R^2 + \left[\frac{2\alpha - 3k}{3}\right] \tag{17}$$

The integration constant has been taken zero for simplicity.

Equation (17) leads to

$$\frac{dR}{\sqrt{R^2+\beta^2}} = adt \tag{18}$$

Where $\beta^2 = \frac{2\alpha - 3k}{2\pi \mathrm{G\,f}}$, $a^2 = \frac{2\pi \mathrm{G\,f}}{3}$

Equation (18) leads to

$$R = \beta \sinh(at+b) \tag{19}$$

Where b is constant of integration

Let at+b=T, so we have

$$R = \beta \sinh T \tag{20}$$

Thus, we have

$$\Lambda = \frac{\alpha}{\beta^2} \cos ech^2 T \tag{21}$$

From Equations (12), (20) and (21), we have

$$8\pi \mathrm{G}\,\rho = 4\pi\,\mathrm{Gf} + 3\mathrm{a}^2 + \left(\frac{3\mathrm{k} - \alpha}{\beta^2} + 3a^2\right)\cos ech^2 T \tag{22}$$

Now Equation (9) leads to

$$8\pi\mathrm{G}\ \left[\dot{\rho} - \mathrm{f}\ \dot{\mathrm{C}}\,\ddot{\mathrm{C}}\ \right] + 8\pi\mathrm{G}\ \left[4\rho\frac{\dot{\mathrm{R}}}{\mathrm{R}} - \frac{3\dot{\mathrm{R}}}{\mathrm{R}}\mathrm{f}\dot{\mathrm{C}}^2\right] + \dot{\Lambda} = 0 \tag{23}$$

Substituting Equation (20), (21) and (22) into equation (23), we have

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{dt}}\dot{\mathrm{C}}^2 + (6a\coth T)\dot{\mathrm{C}}^2 &= \frac{a}{\pi\mathrm{Gf}}\coth T\left\{4\pi\,\mathrm{Gf} + 3\mathrm{a}^2 + \left(\frac{3\mathrm{k}-\alpha}{\beta^2} + 3a^2\right)\cos ech^2 T\right\} + \\ &\frac{1}{4\pi\mathrm{Gf}}\left(\frac{3\mathrm{k}-\alpha}{\beta^2} + 3a^2\right)(-2a\coth T Coseh^2 T) - \frac{2a\alpha}{4\pi\mathrm{Gf}\beta^2}\coth T\cos ech^2 T\end{aligned} \tag{24}$$

Where T=at+b

Equation (24) leads to

$$\frac{\mathrm{d}}{\mathrm{dt}}\dot{\mathrm{C}}^2 + [6a\coth(at+b)]\dot{\mathrm{C}}^2 = 6a\coth(at+b) \tag{25}$$

Equation (25) leads to

$$\dot{C} = 1 \tag{26}$$

and

$$C = t \tag{27}$$

Here we find $\dot{C} = 1,$ which agrees with the value used in the source equation. The creation field C is proportional to time, t.

4. Physical and Geometrical Aspects

Metric (1) for the constrained mentioned earlier, leads to

$$ds^2 = dT^2 - \beta^2 \sinh^2(at+b)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2\right] \tag{28}$$

where k = 0, –1, 1.

The homogeneous mass density ρ, the cosmological constant Λ, the scale factor R, the deceleration parameter, q, and the creation field, C, for the model (28) are given by

$$8\pi G \rho = 4\pi G f + 3a^2 + \left(\frac{3k - \alpha}{\beta^2} + 3a^2\right) \cos ech^2 (at+b) \tag{29}$$

$$\Lambda = \frac{\alpha}{\beta^2} \cos ech^2 (at+b) \tag{30}$$

$$R = \beta \sinh(at+b) \tag{31}$$

$$q = -\frac{\ddot{R}/R}{\dot{R}^2/R^2} = -\tan h^2(at+b) \tag{32}$$

$$C = t \tag{33}$$

Thus $\dot{C}$ = 1 which matches with the result as obtained by Hoyle and Narlikar [10].

5. Conclusion

We find that matter density (ρ) is positive in all the cases (k = 0, –1, 1). The scale factor (R) increases exponentially representing inflationary scenario. The model also represents accelerating universe as deceleration parameter q < 0. The creation field increases with time which matches with the result as obtained by Hoyle and Narlikar [10]. The model (28) passes through a singular event at t = –b/a. This is explained as Creation field exists all the time, so there is a big crunch between t = –b/a to t = –∞ and it passes through a singular state t = –b/a and creation is going on upto t = ∞. The model permits such type of scenario as Creation field exists from $t = -\infty$ to $t = \infty$. The dark energy (Λ) decreases with time. In absence of creation field, $\Lambda \sim 1/t^2$ if a = 1, b = 0 which matches with the result as obtained by Beesham [5].

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