Journal of Rajasthan Academy of Physical Sciences
ISSN : 0972-6306; URL : http://raops.org.in
Vol.18, No.1&2, January-June, 2019, 83-94

ANISOTROPIC VISCOUS FLUID COSMOLOGICAL MODELS IN SAEZ-BALLESTER THEORY OF GRAVITATION

Mahesh Kumar Yadav, Sanjeev Kumar and **Raj Kumar Gangele**
Department of Mathematics, Dr. Harisingh Gour Vishwavidyalaya,
Sagar – 470003 (M.P.) India
Email: yadav1976mk@gmail.com, sanjeev.1501.rajpoot@gmail.com,
rkgangele23@gmail.com

Abstract: In this paper, we investigate Anisotropic Bianchi Type-I viscous fluid cosmological models with field equations proposed by Saez-Ballester (phys. letts. A.113 (1995)). To obtain determinate model, we have used the condition that expansion (θ) is proportional to shear (σ) which leads to $A = B^m$, where m is constant and A, B are metric potentials. The coefficient of bulk viscosity (ζ) is taken as a constant. The physical and geometrical properties of the model are also studied.

Keywords: Saez-Ballester Theory, Viscous Fluid, Bianchi Type I Cosmological Model

1. Introduction

In search of a realistic picture of the early universe, Spatially homogeneous and anisotropic cosmological models have widely been studied in the framework General Relativity. Saez and Ballester [25] have developed a theory in which the metric is coupled with a dimensionless scalar field φ in a simple manner. This φ coupling gives a satisfactory description of the weak fields in which an antigravity regime appears in spite of the dimensionless character of the scalar field. Santhi et al. [29] solved field equations of the Saez-Ballester theory using the hybrid expansion law. Singh et al. [31] observed that the Scalar-tensor theories of gravitation have become a focal point of interests in many areas of gravitational physics and cosmology. Shri Ram et al. [30] and Sahu et al. [28], Reddy et al. ([23], [24]) and Rao et al. ([21], [19], [18], [20]), Pradhan et al. [16] are some authors who have investigated several aspects of Saez-Ballester theory of gravitation. Vinutha et al. [33] proposed that the early stages of evolution of the universe, the presence of perfect fluid with or without radiation are quite important for Spatially homogeneous Bianchi models in Saez-Ballester theory. Dubey et al. [6] proposed to study Bianchi type-V universe with perfect fluid and heat flow in Saez-Ballester scalar-tensor theory of gravitation by considering a law of variation of scale factor as an increasing function of time which yields a time dependent Deceleration Parameter (DP). Recently, Kiran et al. [11] have discussed stationary spherically symmetric one-kink model in Saez-Ballester theory of gravitation. Reddy et al. [24] investigated Spatially homogeneous, anisotropic, and tilted Bianchi type-VI0 model in a new scalar-tensor theory of gravitation proposed by Saez and Ballester (1986) when the source for energy momentum tensor is a bulk viscous fluid containing one dimensional cosmic strings. Rao et al. [19] have obtained Bianchi type-II, VIII and IX DE cosmological models in Saez-Ballester theory of gravitation. Ghate and Sontakke [8] have studied the solutions of Bianchi type-IX universe with variable ω in Saez-Ballester theory of gravitation in the presence and absence of magnetic field of energy ρ_b together with constant deceleration parameter. Katore and Shaikh [10] investigated Bianchi type-I magnetized cosmological model in scalar tensor theory with perfect fluid as a source and the behaviour of models in presence and absence of magnetic field with physical properties are discussed. Santhi et al. [29] have solved Saez-Ballester theory field equations using the hybrid expansion law and to explore the solutions of anisotropic Bianchi Type III space time with magnetized modified holographic Ricci dark energy in the framework of Saez–Ballester theory of gravitation. Shree Ram et al. [30] discussed the law of variation for Hubbles parameter gives a new approach for solving field equations that is quite general and suitable for the description of present day Universe. Mishra et al. [14] constructed anisotropic dark energy cosmological models in the Bianchi-V space-time where the energy momentum tensor consists of two non-interacting fluids namely bulk viscous fluid and dark energy fluid. Pogasian and Vachaspati [15] discussed cosmic microwave background anisotropy from wiggly strings. The equation of state (EoS) parameter of the viscous fluid having value lower than -1 generally considered to be significant in the context of DE cosmology Ade et al.([1], [2]). Mao et al. [13] generalised that the effects of pressure anisotropy can be modeled by an anisotropic viscosity with respect to magnetic field lines. Angular momentum can be transferred by anisotropic viscosity. Bali and Singh [3] discussed if coefficient of bulk viscosity (ξ) is inversely proportional to the expansion (θ) in the model then string cosmological model for Bianchi Type-V space - time is possible. Borkar et al. [4] observed that the scalar function f(t) affected the physical parameters of the universe and our model may have more than three spatial-dimensions in the beginning which is a theoretical evidence pointed out in the geometry of universe. Saha [27] proposed a system of Bianchi type I gravitational fields and nonlinear spinor in presence of viscous fluid. Hecke et al. [9] discussed diffusion tensor images using viscous fluid distribution.

Motivated by the above mentioned studies, we investigate Anisotropic viscous fluid cosmological models in Saez-Ballester theory of gravitation. To get the deterministic scenario, we have assumed $A = B^m$ where m is a constant. Some special cases are also studied.

2. Field Equations

We consider the line element for an anisotropic, spatially homogeneous Bianchi type I metric in the form given by

$$ds^2 = -dt^2 + A^2 dx^2 + B^2(dy^2 + dz^2) \quad (1)$$

When A and B are the function of t.

The energy-momentum for viscous fluid distribution is given by Landau and Lifshitz [12] as

$$T_i^j = (\in + p) v_i v^j + p g_i^j - \eta\left(v_{i;}{}^{j} + v^j{}_{;i} + v^j v^l v_{i;l} + v_i v^l v^j_{;l}\right)$$
$$-\left(\zeta - \frac{2}{3}\eta\right) v^l_{;l}\,(g_i^j + v_i v^j) \quad (2)$$

Where ζ and η are the coefficients of bulk and shear viscosity, p be the pressure and v^i is the flow vector together with

$$g_{ij} v^i v^j = -1 \quad (3)$$

Saez-Ballester Scalar-theory [26] field equation with $G = 1$ and $c = 1$ are

$$R_{ij} - \frac{1}{2} g_{ij} R + \Lambda g_{ij} - \omega\phi^n \left(\phi_{,i}\,\phi_{,j} - \frac{1}{2} g_{ij}\,\phi_{,k}\phi^{,k}\right) = -8\pi \quad (4)$$

Where the scalar field ϕ satisfied the equation

$$2\phi^n \phi^i{}_{;j} + n\phi^{n-1}\,\phi_{,k}\phi^{,k} = 0 \quad (5)$$

and

$$T^{ij}_{;j} = 0 \quad (6)$$

Here n is arbitrary constant and ω is a dimensionless coupling constant. (,) and (;) respectively denote partial and covariant derivative with respect to cosmic time t. Considering the form of energy-momentum tensor for viscous fluid (2), the Einstein's field equation (4), for the Bianchi type-I space-time (1) in Saez-Ballester theory are given as

$$2\frac{B_{44}}{B} + \left(\frac{B_4}{B}\right)^2 - \frac{\omega}{2}\phi^n {\phi_4}^2 + \Lambda = -8\pi p + 8\pi\left(\zeta - \frac{2}{3}\eta\right)\theta + 16\pi\eta\frac{A_4}{A} \quad (7)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{\omega}{2}\phi^n {\phi_4}^2 + \Lambda = -8\pi p + 8\pi\left(\zeta - \frac{2}{3}\eta\right)\theta + 16\pi\eta\frac{B_4}{B} \quad (8)$$

$$2\frac{A_4 B_4}{AB} + \left(\frac{B_4}{B}\right)^2 + \frac{\omega}{2}\phi^n {\phi_4}^2 + \Lambda = 8\pi\epsilon \quad (9)$$

$$\phi_{44} + \left(\frac{A_4}{A} + 2\frac{B_4}{B}\right)\phi_4 + \frac{n}{2}\frac{(\phi_4)^2}{\phi} = 0 \quad (10)$$

Where the suffix 4 denotes ordinary differentiation with respect to t. The expansion θ, shear-scalar σ and coefficient of viscosity ($\eta \propto \theta$) are given as

$$\theta = \frac{A_4}{A} + 2\frac{B_4}{B} \tag{11}$$

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{1}{2}\left(\sum_{i=1}^{3} {H_i}^2 - \frac{\theta^2}{3}\right) \tag{12}$$

$$\eta = l\theta = l\left(\frac{A_4}{A} + 2\frac{B_4}{B}\right) \tag{13}$$

Where l is constant.

3. Solution of the field equations

Here we have five unknowns and four independent equations. Hence we need more conditions to find determinate solutions of the field equations.

3.1 Case: A $= B^m$, where m is constant

Now, from equation (7) to (10), we have

$$2\frac{B_{44}}{B} + \left(\frac{B_4}{B}\right)^2 - \frac{\omega}{2}\phi^n {\phi_4}^2 + \Lambda = -8\pi p + 8\pi\left(\zeta - \frac{2}{3}\eta\right)\theta + 16\pi m\eta\frac{B_4}{B} \tag{14}$$

$$(m+1)\frac{B_{44}}{B} + m^2\left(\frac{B_4}{B}\right)^2 - \frac{\omega}{2}\phi^n {\phi_4}^2 + \Lambda = -8\pi p + 8\pi\left(\zeta - \frac{2}{3}\eta\right)\theta + 16\pi\eta\frac{B_4}{B} \tag{15}$$

$$(2m+1)\left(\frac{B_4}{B}\right)^2 + \frac{\omega}{2}\phi^n {\phi_4}^2 + \Lambda = 8\pi\epsilon \tag{16}$$

$$\phi_{44} + (2+m)\frac{B_4}{B}\phi_4 + \frac{n}{2}\frac{(\phi_4)^2}{\phi} = 0 \tag{17}$$

Now, from equation (15) – equation (14), we have :

$$(m^2 - 1)\left(\frac{B_4}{B}\right)^2 + (m-1)\frac{B_{44}}{B} = 16\pi\eta(1-m)\frac{B_4}{B}$$

$$(m+1)\left(\frac{B_4}{B}\right) + \frac{B_{44}}{B_4} = -16\pi\eta$$

$$(m+1)\left(\frac{B_4}{B}\right) + \frac{B_{44}}{B_4} = -16\pi l(m+2)\left(\frac{B_4}{B}\right)$$

Integrating above equation with respect to t then we have:

$$B^{\{(m+1)+16\pi l(m+2)\}}B_4 = K_1 \tag{18}$$

Where, K_1 is an integration constant. Again Integrating the equation (18) with respect to t then we have :

$$\frac{B^{(m+2)(16\pi l+1)}}{(m+2)(16\pi l+1)} = K_1 t + K_2$$

Where, K_2 is an integrating constant.

$$B = [(m+2)(16\pi l+1)(K_1 t + K_2)]^{\frac{1}{(m+2)(16\pi l+1)}}$$

Also, we conclude that

$$A = [(m+2)(16\pi l+1)(K_1 t + K_2)]^{\frac{m}{(m+2)(16\pi l+1)}}$$

Thus the metric (1) reduced to the form

$$ds^2 = -dt^2 + [(m+2)(16\pi l+1)(K_1 t + K_2)]^{\frac{2m}{(m+2)(16\pi l+1)}} dx^2 + [(m+2)(16\pi l+1)(K_1 t + K_2)]^{\frac{2}{(m+2)(16\pi l+1)}}(dy^2 + dz^2) \quad (19)$$

The quadrature expression for the dimensionless scalar field function ϕ, from eq. (17), is found as

$$\phi = \left[\frac{k}{2}\,\frac{(n+2)}{2}\,\frac{((m+2)(16\pi l+1)(K_1 t + K_2))^{\left(\frac{16\pi l}{16\pi l+1}\right)}}{(m+2)16\pi l} + \frac{(n+2)}{2}\,k'\right]^{\frac{2}{(n+2)}} \quad (20)$$

Where k and k' are constant of integration.

In the particular case, the metric potentials as

$$A = [(m+2)(16\pi l+1)(K_1 t + K_2)]^{\frac{m}{(m+2)(16\pi l+1)}}$$

$$B = [(m+2)(16\pi l+1)(K_1 t + K_2)]^{\frac{1}{(m+2)(16\pi l+1)}}$$

Where k_1, and k_2 are integration constants l is proportional constant and $m \neq -2$ since m is a positive constant.

The proper choice of coordinates and constants (i.e. choosing $k_1 = 1, k_2 = 0$) the Saez-Ballester vaccum model is given by

$$ds^2 = -dt^2 + [(m+2)(16\pi l+1)t]^{\frac{2m}{(m+2)(16\pi l+1)}} dx^2 + [(m+2)(16\pi l+1)t]^{\frac{2}{(m+2)(16\pi l+1)}}(dy^2 + dz^2) \quad (21)$$

The Saez-Ballester scalar field in this model (using Eqs. (21) and (17)) is given by

$$\phi = \left[\frac{k}{2}\ \frac{(n+2)}{2}\ \frac{((m+2)(16\pi l+1)t)^{\left(\frac{16\pi l}{16\pi l+1}\right)}}{(m+2)16\pi l} + \frac{(n+2)}{2}\ k'\right]^{\frac{2}{(n+2)}} \tag{22}$$

Where k and k' are constant of integration.

4 Some geometrical and physical features

The density and pressure for the model (19) are given by

$$8\pi\epsilon = \frac{(2m+1)(k_1)^2}{[(m+2)(16\pi l+1)(k_1 t+k_2)]^2} + \frac{\omega(k_1 k)^2}{2[(m+2)(16\pi l+1)(k_1 t+k_2)]^{\frac{2}{16\pi l+1}}} + \Lambda \tag{23}$$

and

$$8\pi p = \frac{(k_1)^2\left[16\pi l(m+2)^2+2m+1\right]}{[(m+2)(16\pi l+1)(k_1 t+k_2)]^2} + \frac{\omega(k_1 k)^2}{2[(m+2)(16\pi l+1)(k_1 t+k_2)]^{\frac{2}{16\pi l+1}}} + \frac{8\pi k_1}{(16\pi l+1)(k_1 t+k_2)}\left(\varsigma - \frac{2k_1 l}{3(16\pi l+1)(k_1 t+k_2)}\right) - \Lambda \tag{24}$$

The energy conditions are given by Ellis [32] (i) $(\epsilon + p) > 0$ and (ii) $(\epsilon + 3p) > 0$.

The condition (i) lead to

$$\frac{(k_1)^2\left[16\pi l(m+2)^2+4m+2\right]}{[(m+2)(16\pi l+1)(k_1 t+k_2)]^2} + \frac{\omega(k_1 k)^2}{[(m+2)(16\pi l+1)(k_1 t+k_2)]^{\frac{2}{16\pi l+1}}} + \frac{8\pi k_1}{(16\pi l+1)(k_1 t+k_2)}\left(\varsigma - \frac{2k_1 l}{3(16\pi l+1)(k_1 t+k_2)}\right) > 0 \tag{25}$$

And the condition (ii) lead to

$$\frac{(k_1)^2\left[48\pi l(m+2)^2+8m+4\right]}{[(m+2)(16\pi l+1)(k_1 t+k_2)]^2} + \frac{2\omega(k_1 k)^2}{[(m+2)(16\pi l+1)(k_1 t+k_2)]^{\frac{2}{16\pi l+1}}} + \frac{24\pi k_1}{(16\pi l+1)(k_1 t+k_2)}\left(\varsigma - \frac{2k_1 l}{3(16\pi l+1)(k_1 t+k_2)}\right) > 2\Lambda \tag{26}$$

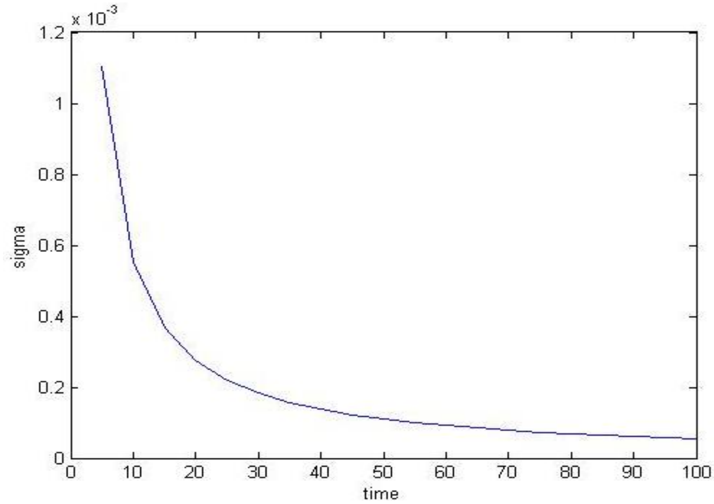
Which give condition on Λ. Thus the reality condition (i) $(\epsilon + p) > 0$, (ii) $(\epsilon + 3p) > 0$ (Ellis[7]) are satisfied and remain the same for viscous fluid when $-2 < m < 2$.

The scalar expansion in this model (19) is

$$\theta = \frac{k_1}{(16\pi l+1)(k_1 t+k_2)} \tag{27}$$

And the shear scalar is

$$\sigma = \frac{(m-1)k_1}{\sqrt{3}(m+2)(16\pi l+1)(k_1 t + k_2)} \tag{28}$$



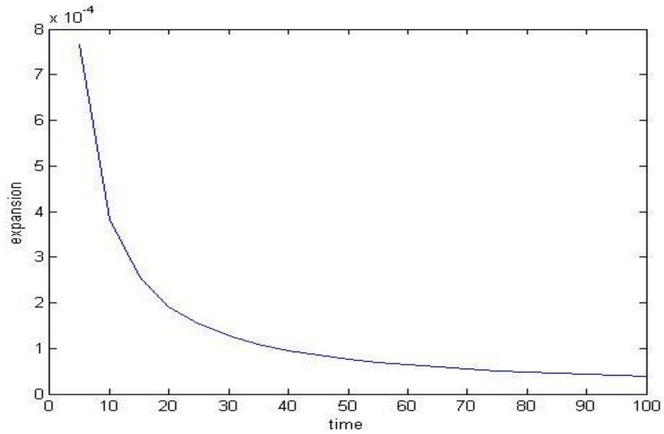
However, if the model (19) isotropizes because

$$\lim_{t\to\infty}\frac{\sigma}{\theta} = 0$$

When $m = 1$. This show that the model isotropizes for $m = 1$.

In the particular case, the proper choice of coordinates and constant (i.e. $k_1 = 1, k_2 = 0$) the scalar expansion in this model (21) is

$$\theta = \frac{k_1}{(16\pi l+1)t} \tag{29}$$



And the shear scalar is

$$\sigma = \frac{(m-1)k_1}{\sqrt{3}(m+2)(16\pi l+1)(K_1 t+K_2)} \tag{30}$$

The density for the model (21) is given by

$$8\pi\epsilon = \frac{(2m+1)}{[(m+2)(16\pi l+1)(t)]^2} + \frac{\omega(k)^2}{2[(m+2)(16\pi l+1)(t)]^{\frac{2}{16\pi l+1}}} + \Lambda \tag{31}$$

And the pressure for the model (21) is given by

$$8\pi p = \frac{[16\pi l(m+2)^2 + 2m+1]}{[(m+2)(16\pi l+1)(t)]^2} + \frac{\omega(k)^2}{2[(m+2)(16\pi l+1)(t)]^{\frac{2}{16\pi l+1}}} + \frac{8\pi}{(16\pi l+1)(t)}\left(\varsigma - \frac{2l}{3(16\pi l+1)(t)}\right) - \Lambda \tag{32}$$

The model (21) has no initial singularity while the matter density, given by (31) and the scalar field ϕ given by (22) possess initial singularities. However as t increases these singularities vanish. The spatial volume of the model given by (22) show the anisotropic expansion of the universe (21) with time. For the model (21) the expansion θ and the shear σ tend to zero as $t \to \infty$ then $\epsilon \to \Lambda, p \to -\Lambda$. The model (21) represents a realistic model.

Also since

$$\lim_{t\to\infty}\left(\frac{\sigma}{\theta}\right) \neq 0 \tag{33}$$

The model (21) does not approach isotropy for a large value of t.

The modern cosmological observations (Ade et al.[1]) constrain the shear to be very small in the post-recombination universe. The early observations of the red-shift for extragalactic sources and in the neighbourhood of our Galaxy today set the limit $\frac{\sigma}{H} \leq 0.3$ (Thorne [34] and Collins [5]).

5. Conclusion

In this paper, we have explored Anisotropic viscous fluid cosmological model in Saez-Ballester theory. Generally, the models are expanding, shearing and non-rotating. The model (19) has a singularity at $t = -\frac{k2}{k1}$ and the scalar expansion θ is decreasing function of time and ultimately become zero for large time, which shows that the universe is expanding with the increase of time. The shear scalar σ decreasing function of time and ultimately $\sigma \to 0$ as $t \to \infty$. Since $\frac{\sigma}{\theta} \neq 0$, thus anisotropy is maintained throughout. However if m = 1, then the model isotropizes.

In particular case, we observe that the model do not approach isotropy for large values of time t. It may be observe that when m = 1, the anisotropy and shear scalar parameters vanish. This show that the universe attains isotropy at late time which confirms present day observations. The Saez-Ballester scalar field increases with time when $m \neq -2$. We observe that the matter density(ε)$\rightarrow 0$ and pressure(p) $\rightarrow 0$ as $t \rightarrow \infty$ thus the matter density and pressure are monotonically decreasing functions of time. The models present structure formation in the universe.

Acknowledgements

The authors are thankful to the Referee for valuable comments and suggestions. One of the authors (MKY) is thankful to UGC, New Delhi for providing support for the present work vide Start up Grant F-30-12/2014(BSR).

References

[1] Ade, P.A.R. (2014). Planck Collaboration XV, Planck 2013 results. XV. CMB power spectra and likelihood, Astronomy Astrophys, A**15**, 571-581.

[2] Ade, P.A.R. (2016). Planck 2015 results. XI. CMB power spectra, likelihoods and robustness of parameters, Astron. and Astrophys., A**11**, 594-593.

[3] Bali, R., and Singh, D.K. (2000). Bianchi Type-V Bulk Viscous Fluid String Dust Cosmological Model in General Relativity, Astrophysics and Space Science, **300**, 387-394.

[4] Borkar, M.S., Ashtankar, N.K. (2013). Bianchi type-I bulk viscous barotropic fluid cosmological model with varying Λ and functional relation on Hubble parameter in self-creation theory of gravitation, American Journal of Modern Physics, **2(5)**, 264-269.

[5] Collins, C.B., Glass, D., Wilkinson, E.N. A. (1980). Exact spatially homogeneous cosmologies, General Relativity and Gravitation, **12(10)**, 805-823.

[6] Dubey, A.S., Khare, R.K. and Pradhan, A. (2013). Anisotropic Bianchi Type-V Cosmological Model with Perfect Fluid and Heat Flow in Saez-Ballester Theory of Gravitation with Variable Deceleration Parameter, ARPN Journal of Science and Technology, **3**, 669- 674.

[7] Ellis G.F.R. (1971). General relativity and cosmology edited by R.K. Sachs, Academic Press, 117.

[8] Ghate, H.R., and Sontakke, A.S. (2014). Bianchi Type-IX Magnetized Dark Energy Model in Saez-Ballester Theory of Gravitation, International Journal of Astronomy and Astrophysics, **4**, 181-191.

[9] Hecke, W.V. et al. (2007). Non rigid coregistration of diffusion tensor images using a viscous a viscous fluid model and mutual information, IEEE Transaction, of medical imaging, **26**, 1598-1612.

[10] Katore, S.D. and Shaikh, A.Y. (2014). Magnetized Cosmological Models in Saez Ballester Theory of Gravitation, Bulg. J. Phys., **41**, 274-290.

[11] Kiran, M., Reddy, D.R.K., Rao, V.U.M. and Baskara Rao, M.P.V. (2014). Stationary spherically symmetric one-kink model in Saez-Ballester theory of gravitation, Astrophys.Space Sci., **356**, 137-139.

[12] Landu, L.D. and Lifshitz, E.M. (1996). Fluid Mechanics, **6**, 1-505.

[13] Mao-Chun Wu, De-Fu Bu, Zhao-Ming Gan and Ye-Fei Yuan (2017). Hot accretion flow with anisotropic viscosity, Astronomy and Astrophysics, **114**, 608-615.

[14] Mishra, B., Ray, P.P. and Pacif, S.K.J. (2018). Anisotropic cosmological models with two fluids, Advances in High Energy Physics, 2018, 1-10.

[15] Pogosian, L. and Vachaspati, T. (1999). Cosmic microwave background anisotropy from wiggly strings, Phys. Rev. D, **60**, 1-10.

[16] Pradhan, A., Singh, A.K. and Chouhan, D.S. (2013). Accelerating Bianchi type-V cosmology with perfect fluid and heat flow in Saez-Ballester theory, Int. J. Theor. Phys., **52**, 266- 278.

[17] Ram, S., Zeyauddin, M. and Singh, C.P. (2009). Bianchi type-V cosmological models with perfect fluid and heat flow in Saez Ballester theory, Parmana Journal of Physics, **72**, 415-427.

[18] Rao, V.U.M., Kumari, G.S.D. and Sireesha, K.V.S. (2011). Anisotropic universe with cosmic strings and bulk viscosity in a scalar tensor theory of gravitation, Astrophys. Space Sci., **335**, 635-644.

[19] Rao, V.U.M., Santhi, M.V. and Vinutha, T. (2008). Exact Bianchi Type-II, VIII and IX Perfect Fluid Cosmological Models in Saez-Ballester Theory of Gravitation, Astrophysics and Space Science, **317**, 27-30.

[20] Rao, V.U.M., Sireesha, K.V.S. and Neelima, D. (2013). Bianchi Type II, VIII, and IX Perfect Fluid Dark Energy Cosmological Models in Saez-Ballester and General Theory of Gravitation, ISRN Astron. and Astrophys., 2013, 1-11.

[21] Rao, V.U.M., Vinutha, T. and Santhi, M.V. (2007). An exact Bianchi type-V cosmological model in Saez-Ballester theory of gravitation, Astrophys. Space Sci., **312**, 189-191.

[22] Reddy, D.R.K. and Naidu, R.L. (2007). Five dimensional string cosmological models in a scalar-tensor theory of gravitation, Astrophys. Space Sci., **307**, 395-398.

[23] Reddy, D.R.K., Naidu, R.L. and Rao, V.U.M. (2006). Axially symmetric cosmic strings in a scalar-tensor theory, Astrophys. Space Sci., **306**, 185-188.

[24] Reddy, D.R.K., Rao, Ch.P., Vidyasagar, T. and Vijaya, R.B. (2013). Anisotropic Bulk Viscous String Cosmological Model in a Scalar-Tensor Theory of Gravitation, Hindawi Publishing Corporation, Advances in High Energy Physics, 2013, 1-5.

[25] Saez, D. and Ballester, V. J. (1985). A simple coupling with cosmological implications, Physics Letters A, **113**, 467-470.

[26] Saez, D. and Ballester, V.J. (1986). A Simple Coupling with Cosmological Implications, Physics Letters A, **133**, 477-486.

[27] Saha, B. (2007). Nonlinear spinor field in Bianchi type-I Universe filled with viscous fluid: numerical solutions, Astrophys Space Sci., **312**, 3-11.

[28] Sahu, S.K., Tole, T.T. and Balcha, M. (2017). Tilted Bianchi type-I wet dark fluid model in Saez and Ballester theory, Indian J Phys, **00**, 1-6.

[29] Santhi, M.V., Rao, V.U.M. and Gusu, D.M. (2018). Magnetized Modified Holographic Ricci Dark Energy Cosmological Model in Saez-Ballester Theory of Gravitation, Prespacetime Journal, **9**, 123-136.

[30] Shri Ram, Zeyauddin, M. and Singh, C.P. (2009). Bianchi type-V cosmological models with perfect fluid and heat flow in Saez-Ballester theory, Pramana journal of physics, **72**, 415-427.

[31] Singh, C.P., Zeyauddin, Mohd. and Shri Ram (2008). Anisotropic Bianchi-V Cosmological Models in Saez-Ballester Theory of Gravitation, Int. J. Mod. Phys. A, **23**, 2719-2731.

[32] Thorne, K.S. (1967). Primordial elements formation, premordial magnetic field and the isotropy of the university, Astropys. J. **148,** 15-55.

[33] Vinutha, T., Rao, V.U.M. and Shanthi, M.V. (2007). An exact Bianchi type-V cosmological model in Saez-Ballester theory of gravitation, Astrophys. Space Sci., **312**, 189-191.