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EFFECT OF SUCTION/INJECTION, RADIATION AND HEAT SOURCE/SINK ON MHD FLOW AND HEAT TRANSFER OVER AN EXPONENTIALLY STRETCHING SHEET

Mamta Goyal and Kuldeep Singh Rathore
Department of Mathematics, University of Rajasthan, Jaipur-302004, India
Email: ksrathore100@gmail.com

Abstract: In this paper, we have investigated a numerical solution for the steady two dimensional laminar flow over an exponentially stretching sheet in the presence of suction-injection with radiative heat flux and internal heat generation. The governing partial differential equations are transformed to a set of ordinary differential equations using similarity transformation and are solved by applying Runge-Kutta method. The fluid is assumed viscous and incompressible. The effects of various quantities of physical interest including Magnetic and Radiation parameters, Suction-injection parameter, Prandtle number, Eckert number and Heat source sink parameter for velocity and temperature distribution are derived. Graphical results for velocity and temperature profiles are presented and discussed.

Key words: MHD, Exponentially stretching sheet, Heat transfer, Radiation, Suction-Injection

1. Introduction

The study of two dimensional boundary layer flows on a continuous stretching sheet has acquired considerable attention due to its numerous applications in industrial, manufacturing processes such as hot rolling, glass fibre, paper production, drawing of plastic films, plastic sheets, metal and polymer processing industries. Sakidas[16] was the first person who studied the boundary layer flow over a stretching sheet whose velocity varies linearly with the distance from a fixed point in the sheet. In 1970, Crane[5] extended this idea for the boundary layer flow caused by a stretching sheet which moves with a velocity varying linearly with the distance from a fixed point. Bidin and Nazar[2] studied the numerical solution of the boundary layer flow over an exponentially stretching sheet with thermal radiation. Magyari and Keller[12] studied the heat and mass transfer in the boundary layers on an exponentially stretching continuous surface. Gupta and Gupta[7], Carragher and Crane[3] have studied heat and mass transfer on stretching sheet. Dutta et al.[6] have investigated temperature field in a flow, over a stretching surface with uniform heat flux. Aboeldahab and Gendy[1] studied radiation effects on

MHD with variable physical property for high temperature difference. Ishak[8] studied the MHD boundary layer flow due to an exponentially stretching sheet with radiation effects. Many other problems on exponentially stretching surface were studied by Raptis et al.[14], Paradha et al.[13] and Sajid & Hayat[15]. Jat and Choudhary[10,11], studied the MHD boundary layer flow near the stagnation point of a stretching sheet. Recently, Jat and Chand [9], investigated MHD flow and heat transfer over an exponentially stretching sheet with viscous dissipation and radiation. In this paper, we have studied the effect of radiation, viscous dissipation, internal heat generation and suction injection on MHD boundary layer flow over an exponentially stretching sheet. The governing partial differential equations are transformed to a set of ordinary differential equations using similarity transformation and are solved by applying Runge-Kutta fourth order method along with shooting technique.

2. Formulation of the Problem

We consider a steady two dimensional laminar flow of a viscous incompressible electrical conducting fluid over a continuous exponentially stretching surface. The x-axis is taken along the stretching surface in the direction of motion and y-axis is perpendicular to it. A uniform magnetic field of strength B_0 is assumed to be applied normal to the stretching surface. The magnetic Reynolds number is taken to be small and therefore the induced magnetic field is neglected. The surface is assumed to be highly elastic and is stretched in the x-direction with a velocity $U = U_0 e^{\frac{x}{L}}$. All the fluid properties are assumed to be constant throughout the motion. Under the usual boundary layer approximations, the governing boundary layer equations by considering the viscous dissipation, radiation effects, heat source/sink effects and perpendicular suction injection are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \kappa\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \sigma B_0^2 u^2 + Q(T - T_\infty) \tag{3}$$

Where u and v are the velocities in the x- and y-directions respectively, ρ is the density of fluid, μ is the dynamic viscosity, $\nu = \frac{\mu}{\rho}$ is the kinematic viscosity, C_p is the specific heat at constant pressure, κ is thermal conductivity of the fluid under consideration, q_r is the radiative heat flux and T is the temperature and $Q = Q_0 e^{\frac{x}{L}}$.

The boundary conditions are:

$$y = 0: \qquad u = U_w = U_0 e^{\frac{x}{L}}, \qquad v = v_0, \qquad T = T_\infty + T_0 e^{\frac{2x}{L}}$$
$$y \to \infty: \qquad u \to 0, \qquad T \to 0 \tag{4}$$

Where U_0, T_0 and L are the reference velocity, temperature and length respectively. In the optically thick limit, the fluid does not absorb its own emitted radiation in which there is no self-absorption, but it does absorb radiation emitted by the boundaries. Cogley et al. [4] showed that in such a case radiative heat flux is given by:

$$\frac{\partial q_r}{\partial y} = 4\left(T - T_\infty\right)\int_0^\infty K_{\lambda w}\left(\frac{de_{b\lambda}}{dT}\right)_w d\lambda = 4I_1\left(T - T_\infty\right) \tag{5}$$

Using equation (5) in equation (3) we get

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \kappa\frac{\partial^2 T}{\partial y^2} - 4I_1(T - T_\infty) + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \sigma B_0^2 u^2 \tag{6}$$

3. Solution of the problem

The equation of continuity (1) is identically satisfied if we choose the stream function ψ such that

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \tag{7}$$

The momentum and energy equations can be transformed into the corresponding ordinary differential equations by introducing the following similarity transformations:

$$\psi\left(x, y\right) = \sqrt{2\nu U_0 L e^{\frac{2x}{L}}}\, f\left(\eta\right)$$

$$\frac{T - T_\infty}{T_0} = e^{\frac{2x}{L}}\theta\left(\eta\right)$$

Where

$$\eta = \sqrt{\frac{U_0}{2\nu L}e^{\frac{2x}{L}}}\, y \tag{8}$$

Then the momentum and energy equations (2) and (6) are transformed to:

$$f^{///} - 2(f^{/})^2 + ff^{//} - Mf^{/} = 0 \tag{9}$$

$$\theta'' - R\theta + \Pr\left(f\theta' - f'\theta + Ec\left(f'' \right)^2 + MEc\left(f' \right)^2 + \gamma\theta \right) = 0 \tag{10}$$

The corresponding boundary conditions are:

$$\eta = 0: \qquad f = S \qquad f' = 1 \qquad \theta = 1$$

$$\eta \to \infty: \qquad f' \to 0 \qquad \theta \to 0 \tag{11}$$

Where prime denotes the differentiation w.r.t. η and dimensionless parameters are:

$M = \dfrac{2\sigma B_0^2 L}{\rho U_0 e^{\frac{x}{L}}}$, (Magnetic parameter)

$Ec = \dfrac{U_0^2}{T_0 c_p}$, (Eckert number)

$\Pr = \dfrac{\mu c_p}{\kappa}$, (Prandtle number)

$R = \dfrac{8 I_1 \nu L}{e^{\frac{x}{L}}}$, (Radiation parameter)

$S = \dfrac{-v_0}{e^{\frac{x}{2L}}} \sqrt{\dfrac{2L}{\nu U_0}}$, (Suction-Injection parameter)

$\gamma = \dfrac{2 Q_0 L}{\rho c_p U_0}$, (Heat source parameter) (12)

The physical quantities of interest are the skin-friction coefficient c_f and the heat transfer rates i.e. Nusselt number Nu are:

$$c_f = \frac{\tau_w}{\frac{\rho U_w^2}{2}} = \frac{\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}}{\frac{\rho U_w^2}{2}}$$

$$\Rightarrow c_f = \frac{2}{\sqrt{\mathrm{Re}}} f''(0) \tag{13}$$

and

$$Nu = \frac{x\left(\frac{\partial T}{\partial y}\right)_{y=0}}{T_w - T_\infty} \tag{14}$$

$$\Rightarrow Nu = -\sqrt{\text{Re}}\,\theta^{/}(0)$$

where

$$\text{Re} = \frac{U_0 L}{\nu} \text{ (Local Reynolds number)} \tag{15}$$

4. Results and discussions

The set of nonlinear ordinary differential equations (9) and (10) with boundary conditions (11) were solved numerically using Runge-Kutta forth order algorithm with a systematic guessing of $f^{//}(0)$ and $\theta^{/}(0)$ by the shooting technique until the boundary conditions at infinity are satisfied. The step size $\Delta\eta = 0.001$ is used while obtaining the numerical solution and accuracy up to the seventh decimal place i.e. 10^{-7}, which is very sufficient for convergence. In this method, we choose suitable finite values of $\eta \to \infty$ say η, which depends on the values of the parameter used. The computations were done by a program which uses a symbolic and computational computer language Matlab.

To access the effects of various parameters on the flow and heat transfer characteristics, the numerical results are presented in figures. The velocity profiles for various values of Magnetic parameter M presented in fig.1 show that the rate of transport is considerably reduced with the increase of M. This is because the increasing value of M tends to the increasing Lorentz force, which produces more resistance to the transport phenomena.

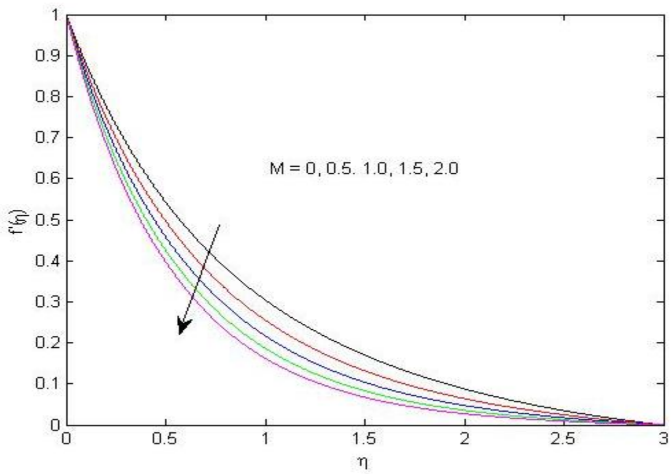

Fig. 1: Velocity profiles against η for various values of Magnetic parameter M (Pr=R=1.0, Ec=γ=S=0)

The temperature profiles for various values of M, R, Pr, keeping Ec and heat source parameter γ as zero and other parameters are fixed to unity, are presented in fig. 2 to 4 respectively. It is observed from the figures that the boundary conditions are satisfied asymptotically in all cases, which supports the accuracy of results obtained.

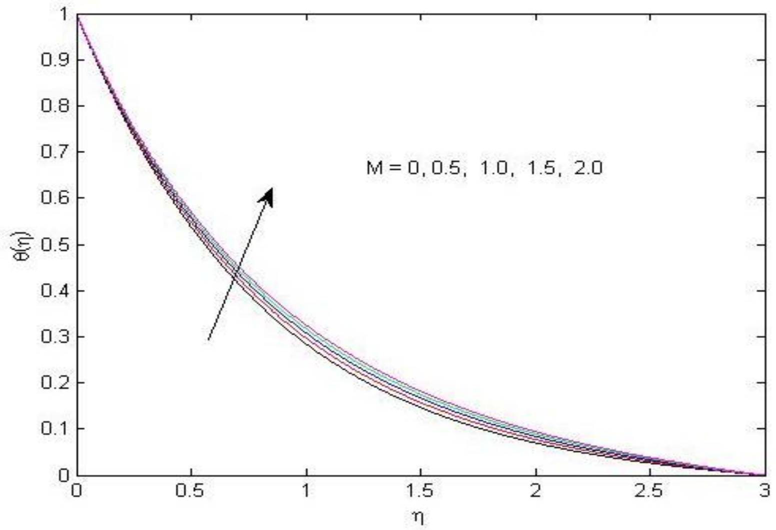

Fig. 2: Temperature profiles against η for various values of Magnetic parameter M (Pr=R=1.0, Ec= γ =S=0)

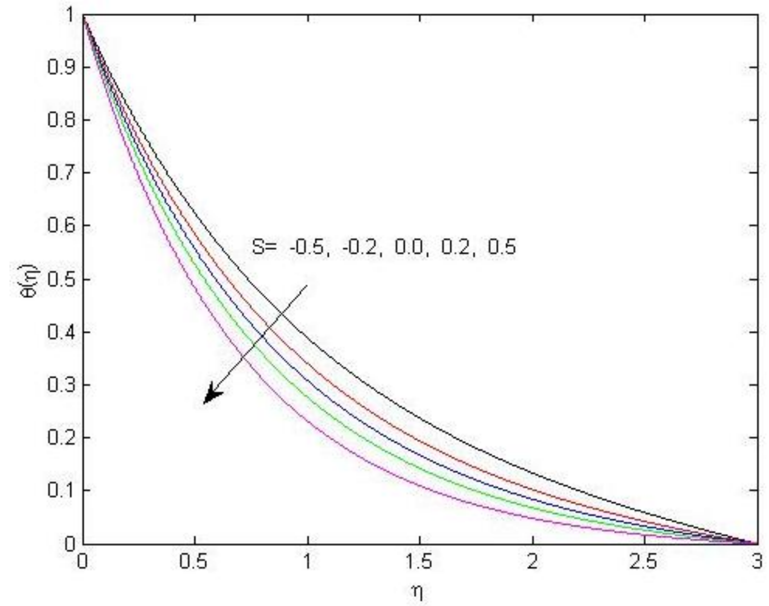

Fig. 3: Temperature profiles against η for various values of Magnetic parameter M (Pr=R=M=1.0, Ec= γ =0)

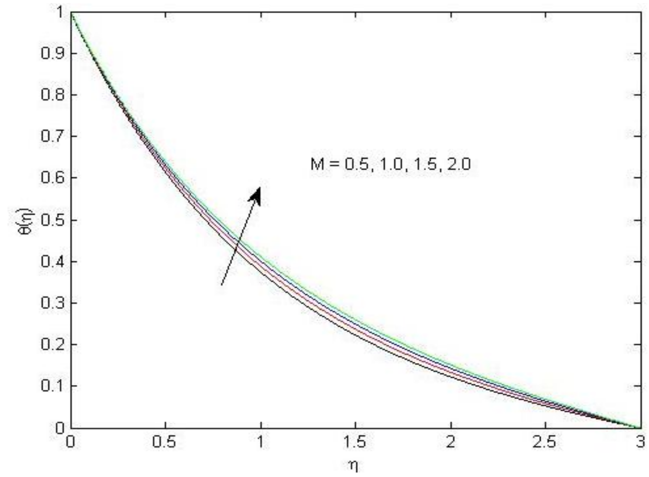

Fig. 4: Temperature profiles against η for various values of Magnetic parameter M (Pr=R=1.0, Ec=γ=S=0)

The variation of velocities and corresponding temperature distributions for applied suction or injection are very important in physical point of view. In confining the generating velocity due to shrinking of sheet and maintaining the boundary layer structure, the mass suction at the sheet is most suitable force. The velocity profiles for various values of suction/injection parameter S are plotted in following figs. Velocity curve increases as applied suction increases whereas velocity curve decreases as injection increases. Thus mass injection acts to increase the momentum boundary layer thickness but suction acts oppositely. The behaviour of temperature profiles for various values of suction injection parameter is presented in the corresponding figures which also show the significant variation in the trend of temperature due to S.

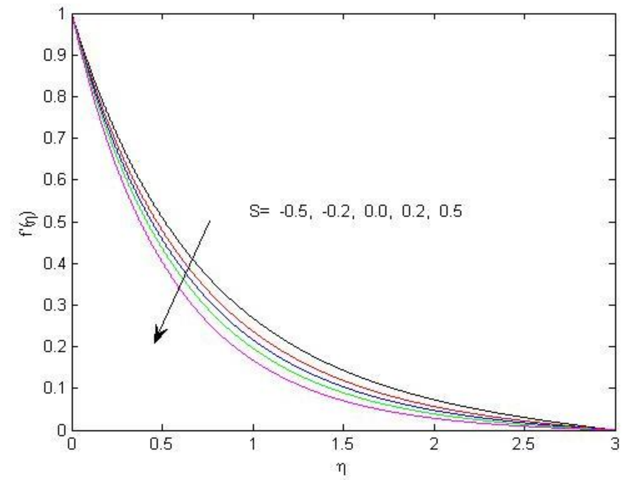

Fig.5: Velocity profile against η for various values of suction-injection parameter S (Pr=R=1.0, Ec=0.2, γ=0.1)

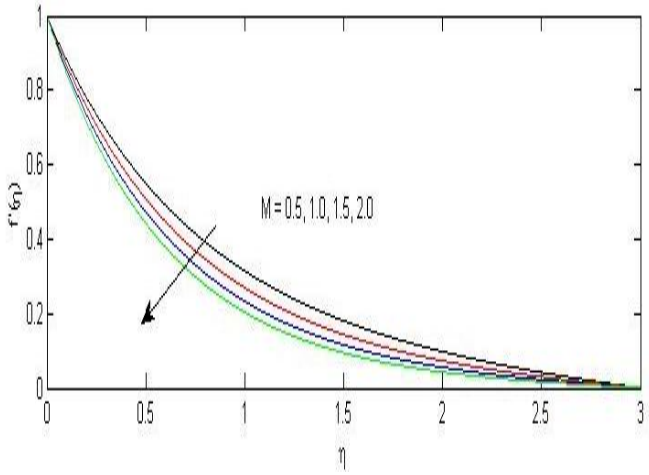

Fig.6: Velocity profile against η for various values of suction-injection parameter S (Pr=R=1.0, Ec=0.5, γ =0.2)

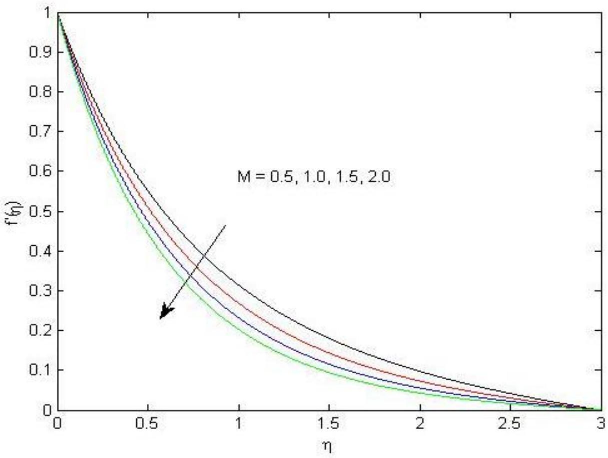

Fig.7: Velocity profile against η for various values of suction-injection parameter S (Pr=R=1.5, Ec=1.0, γ =0.5)

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