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BIANCHI TYPE III BULK VISCOUS MAGNETIZED COSMOLOGICAL MODEL WITH BAROTROPIC FLUID AND DARK ENERGY

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Abstract : The present paper investigates a cosmological model based on the Bianchi type III metric including bulk viscous, magnetized fluid in the presence of dark energy component. The bulk viscosity coefficient is defined as a simple power function of density $\xi = \xi_0 \rho^{\beta}$. A proportional relationship between metric shear (σ) and expansion scalar (θ) is imposed to get a deterministic solution, which leads to a functional relation among metric potentials i.e., $B = C^m$, where $B = B(t)$ and $C = C(t)$. Furthermore, the dark energy component is determined by taking barotropic fluid equation $p = \gamma\rho$; $0 \le \gamma \le 1$. The physical and geometrical properties are discussed in detail.

Keywords : Bianchi Type III, Bulk Viscosity, Magnetized Fluid, Barotropic Equation, Dark Energy.

1. Introduction

Bianchi type-III cosmological models represent spatially homogeneous but directional dependent universes, offering a broader and more flexible framework to study the anisotropic behavior of the early universe and its transition to isotropy over time. The introduction of bulk viscosity in cosmological models refers to the dissipative effects which again plays a significant role in expansion of cosmic fluid and represents a more realistic cosmic fluid behaviour [2]. Bali and Pradhan [3] studied the behavior of bulk viscous fluids in the presence of massive strings within Bianchi type III cosmologies. Mathur et al. [11] explored the role of bulk viscosity and flat potential in the study of inflationary behavior of Bianchi type VIII cosmological framework. Several researchers have investigated the Bianchi Type III metric concerning bulk viscous fluid with string [9, 12, 21].

Furthermore, the cosmological models with the composition of barotropic equation of state; where pressure is proportional to energy density; represent various cosmic epochs, providing a unifying framework that transitions smoothly from early universe to its late-time behavior. They also enhance the investigation through the understanding of anisotropies, dark energy phenomena, and interactions of cosmic fluids with electromagnetic fields and other physical factors. Borkar and Ashtankar [7] analyzed a Bianchi type I universe containing a bulk viscous fluid with barotropic equation of motion, assuming that the cosmological constant varies over time and that the Hubble parameter follows a specific time-dependent form, within the framework of self-creation gravity theory.

Cosmological models that include magnetic fields study the influence of fields on the structure of universe and expansion by causing directional pressures and anisotropies. Xing-Xiang [20] presented the influence of bulk viscosity and magnetic field for Bianchi type-III metric. Bali et al. [4, 5] explored models with bulk viscous barotropic fluid and magnetic field that gives how magnetic forces, internal fluid friction, and pressure-energy relationships work together to influence cosmic growth and smooth out unevenness in space. Chhajed et al. [8] investigated the influence of massive strings within the framework of anisotropic universe having Bianchi type III metric including electromagnetic field. A cylindrically symmetric inhomogeneous cosmological model was developed by Pradhan [14], in which the matter is represented by uniform electromagnetic fluid, and the cosmological constant varies with time.

Models including dark energy are discussed by many researchers. Hembram [10] investigated a model within the framework of general relativity including a varying gravitational constant G(t) and a changing cosmological constant $\Lambda(t)$ with barotropic fluid and magnetic field. Amirhashchi et al. [1] developed a model of the universe filled with string cloud in the context of Bianchi type III metric that incorporates a time-dependent vacuum energy density Λ. Mishra et al. [13] proposed a model with the influence of anisotropy on dark energy and cosmic evolution by treating dark energy as fluid exhibiting directional pressures, consistent with the anisotropic character of the universe. Tyagi et al. [19] studied the impact of a magnetic field and dark energy (Λ) for the Bianchi type-VI_0 universe filled with barotropic fluid.

Singh and Meitei [17] explored their model within the Saez-Ballester scalar-tensor (SBST) gravitational theory involving anisotropic dark energy with a focus on analyzing the deceleration parameter to understand the universe's expansion behavior in this framework. Samanta [15] examined Bianchi type III cosmological models describing anisotropic dark energy within the framework of Lyra geometry and obtained solutions with time-dependent anisotropy and dynamic fluid characteristics. The work of Singh and Rani [16] focused on Bianchi type I cosmological models with varying dark energy parameter in which bulk viscosity plays a major role. Tiwari et al. [18] investigated the interaction and dynamics of two distinct fluid components representing dark energy and barotropic fluid for Bianchi type III space-time. Bayaskar et al. [6] investigated a higher-dimensional Bianchi Type III cosmological model incorporating dark energy and cosmic strings within the framework of f(R) gravity theory.

In this paper we have explored a model with Bianchi type III metric containing bulk viscous, magnetized fluid with dark energy. The bulk viscosity coefficient is modeled as a simple power function of density $\xi = \xi_0 \rho^{\beta}$ [22, 23]. To get deterministic solution we consider proportional relation between shear scalar (σ) and expansion (θ) which establishes a connection between the metric potentials $B = C^m$, where $B = B(t)$ and $C = C(t)$, denoting functions of t alone. Moreover, these metric potentials are written in terms of scalar factor R. Additionally, dark energy component is determined by considering equation of state as $p = \gamma\rho$; $0 \le \gamma \le 1$. The paper also discusses the geometrical and physical attributes of the model for $\beta = 0$ and 1.

2. The Metric and Field Equations

We analyze the line element in an orthogonal configuration for Bianchi type III space-time as follows:

$$ds^2 = -dt^2 + A^2dx^2 + e^{-2x}B^2dy^2 + C^2dz^2 \tag{1}$$

In this context A and B are functions which vary with t and $\sqrt{-g} = ABCe^{-x}$.

The energy-momentum tensor T_i^j, accounting for both a bulk viscous fluid and a magnetic field, is represented as follows:

$$T_i^j = (\bar{p} + \rho)u_i u^j + \bar{p}g_i^j + E_i^j \tag{2}$$

In the above equation $\bar{p}$ represents the effective pressure and p denotes equilibrium pressure. Also they are related to each other by following expression

$$\bar{p} = p - \xi\theta \tag{3}$$

The symbol ξ is termed as bulk viscosity coefficient.

Now considering a co-moving coordinate system such as

$$u^1 = u^2 = u^3 = 0, u^4 = 1 \tag{4}$$

The four-velocity vector, denoted as u^i, fulfills the condition given below

$$g_{ij}u^i u^j = -1 \tag{5}$$

E_i^j denotes the electromagnetic field tensor, which is expressed by

$$E_i^j = \frac{1}{4\pi}\left[F_{ik}F^{jk} - \frac{1}{4}g_i^j F_{kn}F^{kn}\right] \tag{6}$$

As magnetic field is considered in the xy plane, the component along the z-axis is the only one that does not vanish. The Maxwell's equations are expressed as follows

$$\frac{\partial}{\partial x^{j}}\left(F^{ij}\sqrt{-g}\right)=0 \tag{7}$$

Within the framework of geometrized units $8\pi G = c = 1$, the Einstein's field equations including dark energy are given by:

$$R_i^j - \frac{1}{2}Rg_i^j + \Lambda g_i^j = -T_i^j \tag{8}$$

where R_i^j is Ricci tensor and $R = g^{ij}R_{ij}$ is Ricci scalar.

Using (7), the only non vanishing component F_{12} is given by

$$F_{12} = Ke^{-x} \tag{9}$$

where K is a constant.

So the components of E_i^j that survive are determined as follows

$$E_1^1 = E_2^2 = -E_3^3 = -E_4^4 = \frac{K^2}{8\pi A^2 B^2} \tag{10}$$

Using all the above equations, we obtain the Einstein's field equations (8) for metric (1) as follows:

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} + \Lambda = -p + \xi\theta - \frac{K^2}{8\pi A^2 B^2} \tag{11}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} + \Lambda = -p + \xi\theta - \frac{K^2}{8\pi A^2 B^2} \tag{12}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{1}{A^2} + \Lambda = -p + \xi\theta + \frac{K^2}{8\pi A^2 B^2} \tag{13}$$

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{A_4 C_4}{AC} - \frac{1}{A^2} + \Lambda = \rho + \frac{K^2}{8\pi A^2 B^2} \tag{14}$$

$$\frac{B_4}{B} - \frac{A_4}{A} = 0 \tag{15}$$

The symbols A and B, when suffixed with 4, indicate ordinary differentiation with respect to 't'. The scalar expansion is represented by

$$\theta = u^i_{;i} = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \tag{16}$$

The shear scalar σ is given by

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{3}\left(\frac{A_4{}^2}{A^2} + \frac{B_4{}^2}{B^2} + \frac{C_4{}^2}{C^2} - \frac{A_4B_4}{AB} - \frac{B_4C_4}{BC} - \frac{A_4C_4}{AC}\right) \tag{17}$$

Hubble's parameter H can be determined as

$$H = \frac{\theta}{3} = \frac{1}{3}\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) \tag{18}$$

The deceleration parameter is expressed as

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) \tag{19}$$

From equation (15), we get $A = nB$. For simplicity we take $A = B$ (i.e. n=1). The field equations (11) to (14) reduce to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} + \Lambda = -p + \xi\theta - \frac{K^2}{8\pi B^4} \tag{20}$$

$$\frac{2B_{44}}{B} + \frac{B_4{}^2}{B^2} - \frac{1}{B^2} + \Lambda = -p + \xi\theta + \frac{K^2}{8\pi B^4} \tag{21}$$

$$\frac{B_4{}^2}{B^2} + \frac{2B_4C_4}{BC} - \frac{1}{B^2} + \Lambda = \rho + \frac{K^2}{8\pi B^4} \tag{22}$$

3. Solution of Field Equations

Here we are dealing with system comprising three equations (20) to (22), including eight unknown quantities $A, B, C, p, \rho, \xi, \theta$, and Λ. To achieve a deterministic solution we take proportionality relation between expansion scalar θ and shear σ, which results in

$$B = C^m \tag{23}$$

Here m denotes arbitrary constant.

Also the scalar factor R for metric (1) is given by

$$R^3 = ABC \tag{24}$$

Using condition $A = B$ in equations (23) and (24), we obtain

$$B = R^{\frac{3m}{2m+1}} \quad \text{and} \quad C = R^{\frac{3}{2m+1}} \tag{25}$$

Solving equations (20) and (21) with help of equation (25) we have

$$R_{44} + \frac{2R_4^{\,2}}{R} = \frac{K^2(2m+1)}{12\pi(m-1)R^{\frac{10m-1}{2m+1}}} + \frac{(2m+1)}{3(m-1)R^{\frac{4m-1}{2m+1}}} \tag{26}$$

Let us consider $R_4 = f(R)$ and $R_{44} = ff'$ in equation (26), we obtain

$$ff' + \frac{2f^2}{R} = \frac{K^2(2m+1)}{12\pi(m-1)R^{\frac{10m-1}{2m+1}}} + \frac{(2m+1)}{3(m-1)R^{\frac{4m-1}{2m+1}}} \tag{27}$$

Equation (27) leads to

$$\frac{df^2}{dR} + \frac{4f^2}{R} = \frac{K^2(2m+1)}{6\pi(m-1)R^{\frac{10m-1}{2m+1}}} + \frac{2(2m+1)}{3(m-1)R^{\frac{4m-1}{2m+1}}} \tag{28}$$

Solution of (28) is given by

$$f^2 = \frac{K^2(2m+1)^2}{36\pi(m-1)R^{\frac{8m-2}{2m+1}}} + \frac{(2m+1)^2}{9(m^2-1)R^{\frac{2m-2}{2m+1}}} + \frac{L}{R^4} \tag{29}$$

where L is integrating constant.
From equation (29), we have

$$\left(\frac{dR}{dt}\right)^2 = \frac{K^2(2m+1)^2}{36\pi(m-1)R^{\frac{8m-2}{2m+1}}} + \frac{(2m+1)^2}{9(m^2-1)R^{\frac{2m-2}{2m+1}}} + \frac{L}{R^4} \tag{30}$$

Equation (30) on integration gives

$$\int \frac{dR}{\sqrt{\frac{K^2(2m+1)^2}{36\pi(m-1)R^{\frac{8m-2}{2m+1}}} + \frac{(2m+1)^2}{9(m^2-1)R^{\frac{2m-2}{2m+1}}} + \frac{L}{R^4}}} = t + L' \tag{31}$$

where L' acts as the integration constant. The value of R can be extracted from equation (31). Thus, by performing an appropriate transformation of the coordinates, specifically, $R = \tau$, $x = X$, $y = Y$, and $z = Z$, the metric (1) is transformed into

$$ds^2 = \frac{d\tau^2}{\frac{K^2(2m+1)^2}{36\pi(m-1)\tau^{\frac{8m-2}{2m+1}}} + \frac{(2m+1)^2}{9(m^2-1)\tau^{\frac{2m-2}{2m+1}}} + \frac{L}{\tau^4}} + \tau^{\frac{6m}{2m+1}}dX^2$$
$$+ e^{-2x}\tau^{\frac{6m}{2m+1}}dY^2 + \tau^{\frac{6}{2m+1}}dZ^2 \tag{32}$$

4. Physical And Geometrical Characteristics

For the model (32), the expansion θ is given by

$$\theta = \frac{6(m+1)}{(2m+1)}\left(\frac{K^2(2m+1)^2}{36\pi(m-1)\tau^{\frac{12m}{2m+1}}} + \frac{(2m+1)^2}{9(m^2-1)\tau^{\frac{6m}{2m+1}}} + \frac{L}{\tau^6}\right)^{1/2} \tag{33}$$

Also the shear σ is obtained as

$$\sigma = \frac{\sqrt{3}(m-1)}{(2m+1)}\left(\frac{K^2(2m+1)^2}{36\pi(m-1)\tau^{\frac{12m}{2m+1}}} + \frac{(2m+1)^2}{9(m^2-1)\tau^{\frac{6m}{2m+1}}} + \frac{L}{\tau^6}\right)^{1/2} \tag{34}$$

Hubble parameter H is obtained as

$$H = \frac{2(m+1)}{(2m+1)}\left(\frac{K^2(2m+1)^2}{36\pi(m-1)\tau^{\frac{12m}{2m+1}}} + \frac{(2m+1)^2}{9(m^2-1)\tau^{\frac{6m}{2m+1}}} + \frac{L}{\tau^6}\right)^{1/2} \tag{35}$$

Deceleration parameter can be written as

$$q = -1 + \frac{(2m+1)}{4(m+1)}\left\{\frac{\dfrac{m(2m+1)K^2}{8\pi(m-1)\tau^{\frac{12m}{2m+1}}} + \dfrac{2m(2m+1)}{3(m^2-1)\tau^{\frac{6m}{2m+1}}} + \dfrac{6L}{\tau^6}}{\dfrac{K^2(2m+1)^2}{36\pi(m-1)\tau^{\frac{12m}{2m+1}}} + \dfrac{(2m+1)^2}{9(m^2-1)\tau^{\frac{6m}{2m+1}}} + \dfrac{L}{\tau^6}}\right\} \tag{36}$$

The energy density and pressure can be obtained by following equations

$$\rho = \frac{(2m+1)(m+1)K^2}{8\pi(m-1)\tau^{\frac{12m}{2m+1}}} + \frac{(2m+1)}{(m^2-1)\tau^{\frac{6m}{2m+1}}} + \frac{9m(m+2)L}{(2m+1)^2\tau^6} + \Lambda \tag{37}$$

$$p = \frac{(2m-1)(m+1)K^2}{8\pi(m-1)\tau^{\frac{12m}{2m+1}}} - \frac{1}{(m^2-1)T^{\frac{6m}{2m+1}}} + \frac{9m(m+2)L}{(2m+1)^2\tau^6} - \Lambda + \xi\theta \tag{38}$$

To determine Λ, we consider the barotropic fluid condition

$$p = \gamma\rho\,;0 \le \gamma \le 1 \tag{39}$$

Equations (37), (38) and (39) lead to

$$\Lambda(1+\gamma) = \frac{[(2m-1)-(2m+1)\gamma](m+1)K^2}{8\pi(m-1)\tau^{\frac{12m}{2m+1}}} - \frac{[(2m+1)\gamma+1]}{(m^2-1)\tau^{\frac{6m}{2m+1}}} + \frac{9m(m+2)(1-\gamma)L}{(2m+1)^2\tau^6} + \xi\theta \tag{40}$$

In this context, we regard the coefficient of bulk viscosity as a basic power function dependent on energy density i.e.

$$\xi(t) = \xi_0\rho^\beta \tag{41}$$

Where ξ_0 and β are constants.

Case (i) : β = 0, then $\xi(t) = \xi_0 =$ constant

So equation (40) reduces to

$$\Lambda = \frac{[(2m-1)-(2m+1)\gamma](m+1)K^2}{(1+\gamma)8\pi(m-1)\tau^{\frac{12m}{2m+1}}} - \frac{[(2m+1)\gamma+1]}{(1+\gamma)(m^2-1)\tau^{\frac{6m}{2m+1}}} + \frac{9m(m+2)(1-\gamma)L}{(1+\gamma)(2m+1)^2\tau^6} + \frac{\xi_0\theta}{(1+\gamma)} \tag{42}$$

Eliminating Λ in equations (37) and (42), we obtain

$$\rho = \frac{4m(m+1)K^2}{8\pi(m-1)(1+\gamma)\tau^{\frac{12m}{2m+1}}} + \frac{2m}{(m^2-1)(1+\gamma)\tau^{\frac{6m}{2m+1}}} + \frac{18m(m+2)L}{(1+\gamma)(2m+1)^2\tau^6} + \frac{\xi_0\theta}{(1+\gamma)} \tag{43}$$

Also by barotropic condition

$$p = \frac{4m(m+1)K^2\gamma}{8\pi(m-1)(1+\gamma)\tau^{\frac{12m}{2m+1}}} + \frac{2m\gamma}{(m^2-1)(1+\gamma)\tau^{\frac{6m}{2m+1}}}$$

$$+\frac{18m(m+2)L\gamma}{(1+\gamma)(2m+1)^2\tau^6}+\frac{\xi_0\theta\gamma}{(1+\gamma)} \tag{44}$$

For this model the dominant energy condition $p+\rho\geq 0$ leads to

$$\frac{4m(m+1)K^2}{8\pi(m-1)\tau^{\frac{12m}{2m+1}}}-\frac{2m}{\left(m^2-1\right)\tau^{\frac{6m}{2m+1}}}+\frac{18m(m+2)L}{(2m+1)^2\tau^6}+\xi_0\theta\geq 0 \tag{45}$$

Case (ii) : β = 1, then $\xi(t)=\xi_0\rho$

So equations (37), (38) and (42) together can be written as

$$\rho=\frac{1}{\left(1-\frac{\xi_0\theta}{(1+\gamma)}\right)}\left[\frac{4m(m+1)K^2}{8\pi(m-1)(1+\gamma)\tau^{\frac{12m}{2m+1}}}+\frac{2m}{\left(m^2-1\right)(1+\gamma)\tau^{\frac{6m}{2m+1}}}+\frac{18m(m+2)L}{(1+\gamma)(2m+1)^2\tau^6}\right] \tag{46}$$

$$p=\frac{\gamma}{\left(1-\frac{\xi_0\theta}{(1+\gamma)}\right)}\left[\frac{4m(m+1)K^2}{8\pi(m-1)(1+\gamma)\tau^{\frac{12m}{2m+1}}}+\frac{2m}{\left(m^2-1\right)(1+\gamma)\tau^{\frac{6m}{2m+1}}}+\frac{18m(m+2)L}{(1+\gamma)(2m+1)^2\tau^6}\right] \tag{47}$$

Here expansion θ is given by equation (33).

For this model the dominant energy condition $p+\rho\geq 0$ leads to

$$\frac{(1+\gamma)}{(1+\gamma-\xi_0\theta)}\left[\frac{4m(m+1)K^2}{8\pi(m-1)\tau^{\frac{12m}{2m+1}}}+\frac{2m}{\left(m^2-1\right)\tau^{\frac{6m}{2m+1}}}+\frac{18m(m+2)L}{(2m+1)^2\tau^6}\right]\geq 0 \tag{48}$$

For the model (32)

$$\omega=0 \tag{49}$$

The variations of parameters can be shown graphically as follows:

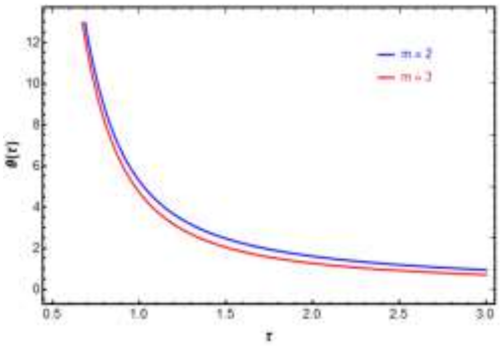

Figure 1: Variation of expansion scalar θ with cosmic time τ for $m = 2,3$.

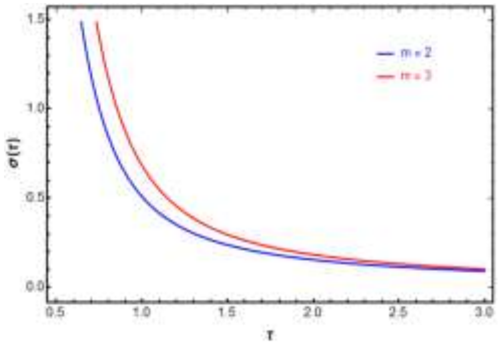

Figure 2: Variation of shear scalar σ with cosmic time τ for $m = 2,3$.

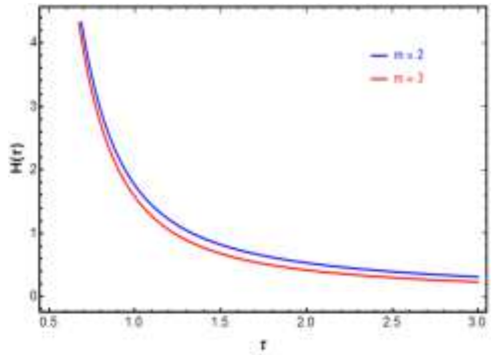

Figure 3: Variation of Hubble parameter H with cosmic time τ for $m = 2,3$.

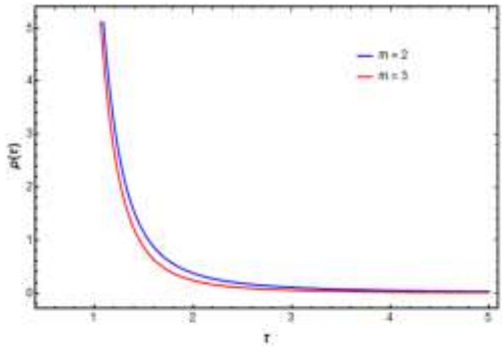

Figure 4: Variation of density ρ with cosmic time τ for $\beta = 0$ $m = 2,3$.

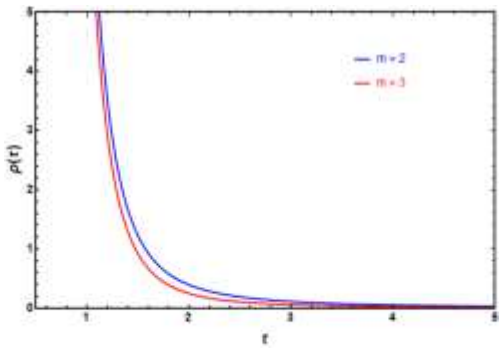

Figure 5: Variation of density ρ with cosmic time τ for $\beta = 1$ $m = 2,3$.

5. Conclusion

The model (32) begins to grow at the big bang $\tau = 0$. Clearly, the domain for 'm', excludes points $m = -1, -\frac{1}{2}$ and 1. So we conclude our results for $m > 1$ As time goes on, the expansion θ slows down, gets closer to zero as $\tau \to \infty$. The model has point type singularity at $\tau = 0$.

Model does not show isotropic behavior for any value of m. We obtained that when $\tau \to \infty$, the ratio of the shear σ and expansion θ tends to a finite value, i.e. $\frac{\sigma}{\theta} = \frac{(m-1)}{2\sqrt{3}(m+1)} \neq 0$ ($m \neq -1$). The deceleration parameter q decreases with time and representing accelerating phase of the universe.

We have taken coefficient of bulk viscosity as a simple power function of energy density $\xi(t) = \xi_0 \rho^{\beta}$, where ξ_0 and β are constants. The cosmological constant, in both cases ($\beta = 0$, 1), decreases with cosmic time for $m > 1$. Additionally, the parameters of these models, including energy density (ρ), pressure (p), shear (σ) and Hubble parameter (H), decrease with time for $m > 1$ and become closer to zero as $\tau \to \infty$. For all values of τ, the condition for energy is satisfied in both cases. As a result, the model obtained is typically demonstrating a universe that is expanding, shearing and non-rotating, with anisotropic characteristics.

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