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CO-AXIAL FLOW MODEL FOR MUCUS TRANSPORT IN CONSTRICTED LUNG AIRWAYS UNDER TIME-VARYING PRESSURE GRADIENT

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Abstract : This paper introduces a three-layer cylindrical co-axial flow model to study mucus transport in constricted airways during mild coughing. Air, mucus and serous flow are treated as incompressible Newtonian fluids under quasi-steady flow conditions. The airflow is assumed to exhibit quasi-steady turbulent behavior while the mucus and serous flow are considered to follow quasi-steady laminar flow. Mucus is considered as a highly viscous fluid whereas serous is less viscous. The influence of a slip parameter accounting for the immotile cilia forming a porous bed is also incorporated into the model. The analysis demonstrates that the flow rates of air, mucus and serous decrease with increasing mucus viscosity. Also, the flow rates of air, mucus and serous increase with higher pressure gradients and greater slip parameter values. Additionally, the study finds that increasing the thickness of airways constriction or the viscosities of mucus and serous leads to a reduction in their flow rates.

Keywords : Quasi-steady state, Turbulent Flow, Time varying pressure gradient, Immotile cilia, Constricted airways.

1. Introduction

The respiratory system relies on the coordinated movement of mucus and serous fluid to protect and maintain airways function. Mucus traps dust, pathogens and other particles while the serous fluid layer provides hydration and aids ciliary action for mucus clearance. In a healthy lung airway, airflow creates sufficient shear forces to move these fluids effectively. However, when the airway is constricted the velocity of airflow and shear stress are reduced disrupting the transport of mucus and serous fluid. This can lead to mucus buildup, decreased airway surface hydration, and an increased risk of obstruction and respiratory infections. Individuals with constriction of airways often experience symptoms such as wheezing, shortness of breath and difficulty breathing, especially during physical activity when airflow demand is higher. In cases of constriction, the altered airflow dynamics severely affect the interaction between mucus and serous fluid. Lower shear forces make it harder for mucus to be cleared and for serous fluid to spread, which disrupts ciliary function and makes breathing problems worse. Scherer et al [17, 18] conducted fluid mechanics experiments on coughing by using turbulent air jets to push air and liquid through a straight tube. By equating the turbulent stress in the air with the viscous force acting on a laminar liquid flow, they found a positive correlation between liquid transport efficiency and the efficiency of turbulent airflow. Agarwal et al. [1, 2] used a constrained simulated cough machine to study mucus gel movement and observed that the presence of serous fluid enhances mucus transport. Kim et al. [7] investigated two-phase flow in pressurized tubes to simulate mucus movement during coughing. Their results provided insight into how mucus travels through narrowed or diseased airways and how airflow dynamics impact transport efficiency. Additionally, they developed a macro-scale flow model to replicate intense airway smooth muscle contraction, highlighting its role in protecting healthy airways. Their model illustrated how adaptive airway constriction serves as a protective mechanism, preventing blockage and promoting respiratory health. Saxena et al. [16], using a four-layer model of mucus transport, found that while increased mucus thickness led to higher flow rates, an excessively thick mucus-serous fluid mixture reduced transport efficiency highlighting the importance of maintaining optimal mucus thickness. Chitra and Radhakrishnan [3] employed a cylindrical model to examine mucociliary transport in the trachea, showing that elevated serous layer viscosity impairs mucus movement. They also observed that there is an optimal mucus viscosity for effective clearance, as further increases beyond a certain point have minimal impact on transport. Kumar et al. [12] investigated mucus movement across varying airway diameters and pressure gradients. They have found that transport efficiency decreased as airway constriction became more severe. The study on Boundary layer theory is given by Schlichting [19], the influence of airway surface liquid by Zahm et al. [28].

In this paper, we aim to investigate a three-layer cylindrical co-axial flow in air, mucus and serous region in the constricted airways. In this model, we assume that the air flow in the central core lumen is of quasi-steady and turbulent type whereas mucus flow surrounding the core lumen is assumed as quasi steady state and laminar. We took into account the impact of the slip parameter caused by the immotile cilia, which form a porous bed in contact with epithelial walls. Here, we use the following assumptions, which have been employed in previous researches by other investigators:

a. Air, mucus and serous flow are co-axial and symmetrical around the central line of the airways.

b. The applied pressure gradient is considered a time-varying function that represents mild coughing.

c. Due to high shear rates during mild coughing, mucus is regarded as an incompressible Newtonian fluid [29].

d. Airflow is assumed turbulent during coughing [18].

e. Both mucus and serous flows are assumed to be laminar and quasi-steady during coughing.

f. The immotile cilia bed which is assumed saturated with watery serous fluid and it is assumed that in this layer, the flow is governed by Darcy's law, imposing slip parameter at the interface of the porous bed and mucus in larger airways.

2. Mathematical Model

This study considers simultaneous and co-axial layers of air, mucus and serous fluid flowing through a circular tube due to time dependent pressure gradient simulating mucus transport in airways due to mild cough as shown in Fig.1. In the central core air is assumed to flow under quasi-steady turbulent condition due to instantaneous pressure gradient caused by mild cough. The mucus layer surrounding this circular core and serous layer surrounding the mucus layer is assumed to flow under quasi-steady laminar conditions. The model also takes into account the influence of a slip parameter induced by immotile cilia in the serous layer that form a porous bed in contact with the epithelial wall. The constriction attaches to the wall penetrating into serous layer is sinusoidal. Smooth muscles attached to the wall can cause the serous layer to contract and tighten in pathological conditions.

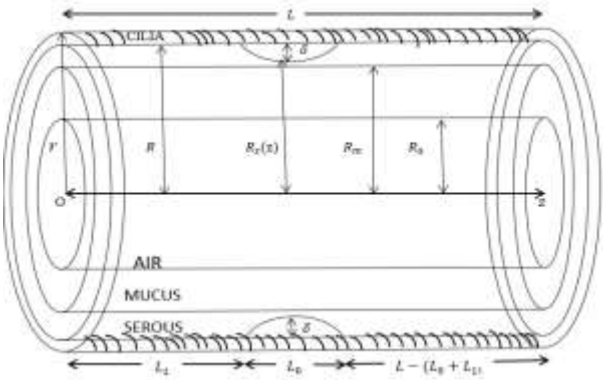


Figure 1: Circular tube geometry for mucus transport in constricted airways.

The radius of the cylindrical tube is defined as [21, 27]

$$\frac{R_s(z)}{R} = \begin{cases} 1 - \frac{\delta}{2R}\{1 + cos\frac{2\pi}{L_0}\left(z - L_1 - \frac{L_0}{2}\right)\}, & L_1 \le z \le L_1 + L_0 \\ 1, & 0 \le z \le L_1 \; and \; L_1 + L_0 \le z \; \le L \end{cases} \quad (1)$$

where R is the radius of circular tube, $R_s(z)$ is the radius of circular tube in constricted area, δ (<< $R_s(z)$) is the thickness of constriction which is sinusoidal.

Let, $a = R - \frac{\delta}{2}$ and $b = \frac{\delta}{2}$ then equation (1) becomes:

$$R_s(z) = a - bcos\frac{2\pi}{L_0}\left(z - L_1 - \frac{L_0}{2}\right)$$

The equation governing the mucus and serous flow under quasi-steady state in a circular tube can be written as follows:

Region I: Air flow region $(0 \le r \le R_a)$

$$\frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}(r\tau_a) \tag{2}$$

$$\tau_a = -\rho_a\, l_a^2 \left(-\frac{\partial u_a}{\partial r}\right)^2 \tag{3}$$

Region I: Mucus Region $(R_a \le r \le R_m)$

$$\frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}(r\tau_m) \tag{4}$$

$$\tau_m = \mu_m \frac{\partial u_m}{\partial r} \tag{5}$$

Region II: Serous Region $\left(R_m \le r \le R_s(z)\right)$

$$\frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}(r\tau_s) \tag{6}$$

$$\tau_s = \mu_s \frac{\partial u_s}{\partial r} \tag{7}$$

where z is the axial coordinate along the tube axis which is in the flow direction and r is the radial coordinate in the radial direction which is perpendicular to the fluid flow, R_a is the thickness up to air-mucus interface, R_m is the thickness up to mucus-serous interface, p is the mean pressure that is constant across the two layers, τ_m and τ_s are the mean shear stress across mucus and serous region, u_a u_m and u_s are the mean velocity components of the air, mucus and serous in the direction of z and μ_m, μ_s and $\mu_a$are the viscosities of mucus , serous and air respectively.

By using Prandtl mixing length theory, l_a is assumed as

$$l_a = l_0(R-r) \tag{8}$$

where l_0 is constant and determined experimentally [17].

The formation of porous bed by immotile cilia during mild coughing or forced expiration causes slipperiness at the boundary $r = R_s(z)$. Therefore,

Boundary Conditions:

$$\frac{\partial u_a}{\partial r} = 0, \qquad r = 0 \tag{9}$$

$$u_s = -\beta\tau_s,\ r = R_s(z) \tag{10}$$

Matching Conditions:

$$u_a = u_m, \qquad r = R_a \quad , u_m = u_s, \qquad r = R_m \tag{11}$$

$$\tau_a = \tau_m, \qquad r = R_a \quad , \tau_m = \tau_s, \qquad r = R_m \tag{12}$$

The negative sign in equation (10) arises from the negative value of shear stress, τ_s, within the serous layer. This reflects the abrupt constriction of airway smooth muscles, resulting in a rapid variation in the velocity gradient. The negative sign in the shear stress expression signifies that the stress acts in opposition to the airflow direction, a critical factor in analyzing the mechanical behavior during bronchoconstriction. It is also important to note that β represents the slip parameter at the interface between the mucus layer and the immotile cilia which are saturated with the watery serous fluid and form a porous bed in contact with the airway epithelial surface. Continuity of velocity and shear stress across the air-mucus and mucus-serous interfaces is maintained through equations (11) and (12), respectively [20].

Since during mild coughing, the pressure gradient in the lung is time dependent. Therefore, we may assume that

$$-\frac{\partial p}{\partial z} = P = P_0 f(t) \tag{13}$$

where t represents time and P_0 is constant affected by the intensity of the mild cough. The magnitude of the mild cough is dependent upon the intensity of the turbulence caused by the cough. A cough that is more intense causes a corresponding increase in flow rates. If there is not even a slight cough, P is zero everywhere. However, mucus transport continues to occur because of the mean velocity of the cilia beating [15]. The function f(t) in equation (13) taken from Kumar et.al [11] which characterizes the mild cough and is defined as:

$$f(t) = \begin{cases} \frac{3t^2}{8T_m}\left(1 - \frac{2t}{3T_m}\right), & 0 \le t \le T_m \\ \frac{9t}{32}\left(1 - \frac{9t}{10T}\right)^2, & T_m \le t \le \frac{T}{\alpha} \\ 0, & \frac{T}{\alpha} \le t \end{cases} \tag{14}$$

where, $\alpha = 0.9$ and $T_m = 0.011$ and T is the cough duration.

To simplify the analysis, we can assume the cough duration T spans from 0.030 seconds to 0.035 seconds. The graphical depiction of equation (14) is illustrated in Figure 2.

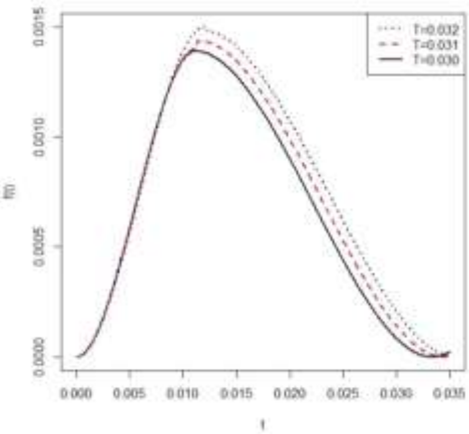


Figure 2: Graphical Representation of $f(t)$ for various value of T.

3. Analytical Solution

Solving equations (1)-(7) by using boundary and matching conditions (9)-(12), the stress and velocity components in air and mucus layers are computed which are given below:

$$\tau_a = \tau_m = \tau_s = -\frac{Pr}{2} \tag{15}$$

$$u_a = \frac{P}{4\mu_m}(R_m^2 - R_a^2) + \frac{P}{4\mu_s}(R_s^2(z) - R_m^2) + \frac{\beta P R_s(z)}{2} + \sqrt{\frac{PR}{2\rho_a l_0^2}}\left[\log\left|\frac{\sqrt{R}+\sqrt{r}}{\sqrt{R}-\sqrt{r}}\right| - \log\left|\frac{\sqrt{R}+\sqrt{R_a}}{\sqrt{R}-\sqrt{R_a}}\right| - \frac{2(\sqrt{r}-\sqrt{R_a})}{\sqrt{R}}\right] \tag{16}$$

$$u_m = \frac{P}{4\mu_m}(R_m^2 - r^2) + \frac{P}{4\mu_s}(R_s^2(z) - R_m^2) + \frac{\beta P R_s(z)}{2} \tag{17}$$

$$u_s = \frac{P}{4\mu_s}(R_s^2(z) - r^2) + \frac{\beta P R_s(z)}{2} \tag{18}$$

The volumetric flow rates in the three layers (air, mucus and serous) can be defined as follows:

$$Q_a = \int_0^{R_a} 2\pi u_a dr, \quad Q_m = \int_{R_a}^{R_m} 2\pi u_m dr \quad, Q_s = \int_{R_m}^{R_s(z)} 2\,\pi r u_s dr \tag{19}$$

Substituting the values of u_a from (16), u_m from (17) and u_s from (18) in equation (19) respectively, we get

$$\frac{Q_a}{2\pi} = \frac{PR_a^2}{8\mu_m}(R_m^2 - R_a^2) + \frac{PR_a^2}{8\mu_s}(R_s^2(z) - R_m^2) + \frac{P\beta R_a^2 R_s(z)}{4} + \sqrt{\frac{PR^5}{8\rho_a l_0^2}}\left[\log\left|\frac{\sqrt{R}-\sqrt{R_a}}{\sqrt{R}+\sqrt{R_a}}\right| - 2\sqrt{\frac{R_a}{R}}\left(1 + \frac{R_a}{3R} + \frac{R_a^2}{5R^2}\right)\right] \tag{20}$$

$$\frac{Q_m}{2\pi} = \frac{P}{16\mu_m}\left(R_m^2 - {R_a}^2\right)^2 + \frac{P}{8\mu_s}(R_s^2(z) - R_m^2)\left(R_m^2 - {R_a}^2\right) + \frac{P\beta R_s(z)}{4}\left(R_m^2 - {R_a}^2\right) \tag{21}$$

$$\frac{Q_s}{2\pi} = \frac{P}{16\mu_s}\left(R_s^2(z) - {R_m}^2\right)^2 + \frac{P\beta R_s(z)}{4}(R_s^2(z) - R_m^2) \tag{22}$$

To calculate the pressure drop in each layer, we understand from the equation of continuity that Q_a, Q_m and Q_s are constants. Therefore, from (20), (21) and (22) we have

$$-\frac{\partial p}{\partial z} = \frac{64\mu_s^2 K_3^2}{R_a^4\left(R_s^2(z) + K_1 R_s(z) - K_2\right)^2} + \frac{Q_a}{\pi}\left[\frac{8\mu_s}{R_a^2\left(R_s^2(z) + K_1 R_s(z) - K_2\right)}\right] \tag{23}$$

$$-\frac{\partial p}{\partial z} = \frac{Q_m}{2\pi K_4\left(R_s^2(z) + K_1 R_s(z) - K_5\right)} \tag{24}$$

$$-\frac{\partial p}{\partial z} = \frac{Q_s}{2\pi K_6\left(R_s^2(z) - R_m^2\right)\left(R_s^2(z) + 2K_1 R_s(z) - R_m^2\right)} \tag{25}$$

where,

$$K_1 = 2\beta\mu_s, K_2 = R_m^2\left(1 - \frac{\mu_s}{\mu_m}\right) + R_a^2,\ \ K_3 = \sqrt{\frac{R^5}{8\rho_a l_0^2}}\left[\log\left|\frac{\sqrt{R}-\sqrt{R_a}}{\sqrt{R}+\sqrt{R_a}}\right| - 2\sqrt{\frac{R_a}{R}}\left(1 + \frac{R_a}{3R} + \frac{R_a^2}{5R^2}\right)\right],$$

$K_4 = \frac{R_m^2 - R_a^2}{8\mu_s}, K_5 = R_m^2\left(1 - \frac{\mu_s}{2\mu_m}\right) + R_a^2, K_6 = \frac{1}{16\mu_s}$

Replacing $R_s(z)$ by R for non-constricted regions ($0 \le z \le L_1$ and $L_1 + L_0 \le z \le L$). Then the pressure gradient for non-constricted regions becomes

$$-\frac{\partial p}{\partial z} = \frac{64\mu_s^2 K_3^2}{R_a^4(R^2+K_1R-K_2)^2} + \frac{Q_a}{\pi}\left[\frac{8\mu_s}{R_a^2(R^2+K_1R-K_2)}\right] \quad (26)$$

$$-\frac{\partial p}{\partial z} = \frac{Q_m}{2\pi K_4(R^2+K_1R-K_5)} \quad (27)$$

$$-\frac{\partial p}{\partial z} = \frac{Q_s}{2\pi K_6(R^2-R_m^2)(R^2+2K_1R-R_m^2)} \quad (28)$$

Since the pressure is present only at two ends of the tube i.e., $p = p_0$ at $z = 0$, $p = p_L$ at $z = L$. Then, we define the pressure drop as $\Delta P = p_0 - p_L$. Now, integrating equations (23) and (26), we get

$$\Delta P = -\int_0^L d\,p \quad = \int_0^{L_1} \frac{64\mu_s^2 K_3^2}{R_a^4(R^2+K_1R-K_2)^2} + \frac{Q_a}{\pi}\left[\frac{8\mu_s}{R_a^2(R^2+K_1R-K_2)}\right] dz + \int_{L_1}^{L_1+L_0} \frac{64\mu_s^2 K_3^2}{R_a^4\left(R_s^2(z)+K_1R_s(z)-K_2\right)^2} + \frac{Q_a}{\pi}\left[\frac{8\mu_s}{R_a^2\left(R_s^2(z)+K_1R_s(z)-K_2\right)}\right] dz$$

$$+ \int_{L_1+L_0}^{L} \frac{64\mu_s^2 K_3^2}{R_a^4(R^2+K_1R-K_2)^2} + \frac{Q_a}{\pi}\left[\frac{8\mu_s}{R_a^2(R^2+K_1R-K_2)}\right] dz$$

Putting the value of $R_s(z)$ from (1) in above equation, we get

$$\Delta P = \frac{64\mu_s^2 K_3^2(L-L_0)}{R_a^4(R^2+K_1R-K_2)^2} + \frac{64\mu_s^2 K_3^2 L_0}{R_a^4(n-m)^2}\left[\frac{a+m}{((a+m)^2-b^2)^{3/2}} + \frac{a+n}{((a+n)^2-b^2)^{3/2}}\right]$$
$$+ \frac{128\mu_s^2 K_3^2 L_0}{R_a^4(n-m)^3}\left[\frac{1}{((a+n)^2-b^2)^{1/2}} - \frac{1}{((a+m)^2-b^2)^{1/2}}\right]$$
$$+ \frac{Q_a}{\pi}\frac{8\mu_s(L-L_0)}{R_a^2(R^2+K_1R-K_2)} + \frac{Q_a}{\pi}\frac{8\mu_s L_0}{R_a^2(n-m)}\left[\frac{1}{((a+m)^2-b^2)^{1/2}} - \frac{1}{((a+n)^2-b^2)^{1/2}}\right] \quad (29)$$

where $m = \frac{K_1}{2} + \sqrt{K_2 + \frac{K_1^2}{4}}$ and $n = \frac{K_1}{2} - \sqrt{K_2 + \frac{K_1^2}{4}}$

Let

$$X_1 = \frac{8\mu_s(L-L_0)}{R_a^2(R^2+K_1R-K_2)} + \frac{Q_a}{\pi}\frac{8\mu_s L_0}{R_a^2(n-m)}\left[\frac{1}{((a+m)^2-b^2)^{1/2}} - \frac{1}{((a+n)^2-b^2)^{1/2}}\right]$$

$$Y_1 = \frac{64\mu_s^2 K_3^2(L-L_0)}{R_a^4(R^2+K_1R-K_2)^2} + \frac{64\mu_s^2 K_3^2 L_0}{R_a^4(n-m)^2}\left[\frac{a+m}{((a+m)^2-b^2)^{3/2}} + \frac{a+n}{((a+n)^2-b^2)^{3/2}}\right] +$$

$$\frac{128\mu_m^2 K_3^2 L_0}{R_a^4(n-m)^3}\left[\frac{1}{((a+n)^2-b^2)^{1/2}} - \frac{1}{((a+m)^2-b^2)^{1/2}}\right]$$

then the equation (29) becomes

$$\Delta P = Y_1 - \frac{Q_a}{\pi}X_1$$

The volumetric flow rate in air region i.e. Q_a can be found as follows :

$$Q_a = \pi\left(\frac{\Delta P}{X_1} - \frac{Y_1}{X_1}\right) \quad (30)$$

Similarly integrating equations (24) and (27), we get

$$\Delta P = -\int_0^L d\,p \quad = \int_0^{L_1} \left[\frac{Q_m}{2\pi K_4 (R^2 + K_1 R - K_5)}\right] dz + \int_{L_1}^{L_1+L_0} \left[\frac{Q_m}{2\pi K_4 \left(R_s^2(z) + K_1 R_s(z) - K_5\right)}\right] dz$$

$$+ \int_{L_1+L_0}^{L} \left[\frac{Q_m}{2\pi K_4 (R^2 + K_1 R - K_5)}\right] dz$$

Putting the value of $R_s(z)$ from (1) in above equation, we get

$$\Delta P = \frac{Q_m}{2\pi K_4}\left\{\frac{(L-L_0)}{(R^2+K_1R-K_5)} + \frac{L_0}{(n^*-m^*)}\left[\frac{1}{((a+m^*)^2-b^2)^{\left(\frac{1}{2}\right)}} - \frac{1}{((a+n^*)^2-b^2)^{\left(\frac{1}{2}\right)}}\right]\right\}$$

where, $m^* = \frac{K_1}{2} + \frac{\sqrt{K_1^2+4K_5}}{2}$ and $n^* = \frac{K_1}{2} - \frac{\sqrt{K_1^2+4K_5}}{2}$.
The volumetric flow rate of mucus i.e.; Q_m can be found as follows:

$$Q_m = \frac{2\pi K_4 \Delta P}{\left\{\frac{(L-L_0)}{(R^2+K_1R-K_5)} + \frac{L_0}{(n^*-m^*)}\left[\frac{1}{((a+m^*)^2-b^2)^{\left(\frac{1}{2}\right)}} - \frac{1}{((a+n^*)^2-b^2)^{\left(\frac{1}{2}\right)}}\right]\right\}} \tag{31}$$

Similarly, integrating equation (25) and (28), we get

$$\Delta \mathrm{P} = -\int_0^L d\,p \quad = \int_0^{L_1} \frac{Q_s\,dz}{2\pi K_6 (R^2 - R_m^2)(R^2 + 2K_1 R - R_m^2)} + \int_{L_1}^{L_1+L_0} \frac{Q_s\,dz}{2\pi K_6 \left(R_s^2(z) - R_m^2\right)\left(R_s^2(z) + 2K_1 R_s(z) - R_m^2\right)}$$

$$+ \int_{L_1+L_0}^{L} \frac{Q_s\,dz}{2\pi K_6 (R^2 - R_m^2)(R^2 + 2K_1 R - R_m^2)}$$

Putting the value of $R_s(z)$ from (1) in above equation, we get

$$\Delta P \quad = \frac{Q_s}{2\pi K_6}\frac{(L-L_0)}{(R^2-R_m^2)(R^2+2K_1R-R_m^2)} + \frac{Q_s}{2\pi K_6}\frac{L_0}{2R_m}\left[\frac{((a-R_m)^2-b^2)^{\left(-\frac{1}{2}\right)}}{(u+R_m)(v+R_m)} - \frac{((a+R_m)^2-b^2)^{\left(-\frac{1}{2}\right)}}{(u-R_m)(v-R_m)}\right]$$

$$+ \frac{Q_s}{2\pi K_6}\frac{L_0}{(u-v)}\left[\frac{((a+v)^2-b^2)^{\left(-\frac{1}{2}\right)}}{(v^2-R_m^2)} - \frac{((a+u)^2-b^2)^{\left(-\frac{1}{2}\right)}}{(u^2-R_m^2)}\right]$$

where, $u = K_1 + \sqrt{K_1^2 + R_m^2}$ and $v = K_1 - \sqrt{K_1^2 + R_m^2}$.
The volumetric flow rate of serous i.e.; Q_s can be found as follows:

$$Q_s = \frac{\pi K_6 \Delta P}{\left\{\frac{(L-L_0)}{(R^2-R_m^2)(R^2+2K_1R-R_m^2)} + \frac{L_0}{2R_m}\left[\frac{((a-R_m)^2-b^2)^{\left(-\frac{1}{2}\right)}}{(u+R_m)(v+R_m)} - \frac{((a+R_m)^2-b^2)^{\left(-\frac{1}{2}\right)}}{(u-R_m)(v-R_m)}\right]\frac{L_0}{(u-v)}\left[\frac{((a+v)^2-b^2)^{\left(-\frac{1}{2}\right)}}{(v^2-R_m^2)} - \frac{((a+u)^2-b^2)^{\left(-\frac{1}{2}\right)}}{(u^2-R_m^2)}\right]\right\}} \tag{32}$$

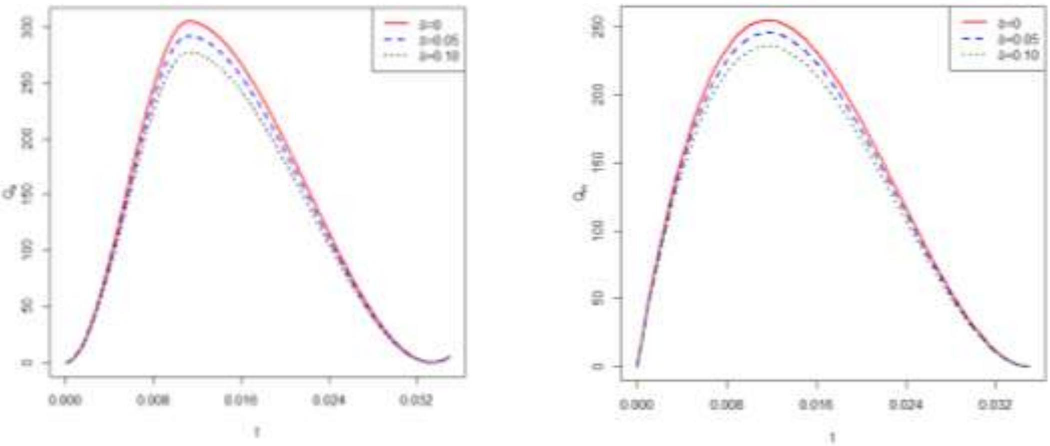
4. Results and Discussion

To investigate the impact of different model parameters on the flow rates of air, mucus and serous, the values of Q_a, Q_m and Q_s as provided by equations (30), (31) and (32) were calculated using the following dataset [22, 26]:

$R = 90.00 \times 10^{-2}$ cm, $R_m = 38.45 \times 10^{-2}$ cm, $R_a = 31.45 \times 10^{-2}$ cm, $t = 0 - 0.035$ sec, T = 0.035 sec, L = 1.0 cm, L_0= 0.5 cm, l_0= 0.40 cm, β= 0 − 0.10 gm cm^2 sec, δ = 0 − 0.1 cm,

μ_m = 1.00 − 10.00 poise, $P_0 = (1 - 10) \times 10^5$ gm cm^{-2} sec^{-2}, μ_a= 0.0002 poise, $\rho_a = 1 \times 10^{-3} gm\ cm^{-3}$.

The variations in volumetric flow rates Q_a , Q_m and Q_s with respect to time t are shown in following figures:



Figures 3: Variations of Q_a and Q_m with t for different values of δ

Figures 3 illustrate the impact of time on the flow rates of air and mucus, for fixed values of $T = 0.030$ sec, $L = 1$ cm, $L_0 = 0.5$ cm, $R = 90.00 \times 10^{-2}$ cm, $R_a = 31.45 \times 10^{-2}$ cm, $\mu_m = 1$ poise, $\beta = 0.05$ gm cm^{2}sec, $P_0 = 1 \times 10^5$ gm cm^{-2} sec^{-2}and $\mu_a = 0.0002$ poise for different values of δ. It is observed that the volumetric flow rates of air and mucus decrease with the increase in thickness of constriction. These results in line with the findings of Kumar et al. [12] and Chitra and Shabana [4].

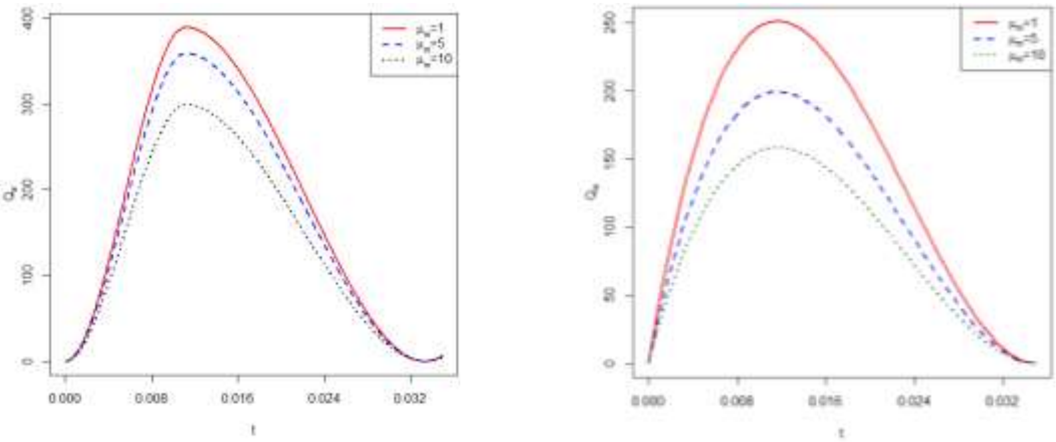


Figure 4: Variations of Q_a and Q_m with t for different values of μ_m

Figure 4 show the impact of time on air and mucus flow rates for fixed values of $T = 0.030$ sec, $L = 1$ cm, $L_0 = 0.5$ cm, $R = 90.00 \times 10^{-2}$ cm, $R_a = 31.45 \times 10^{-2}$ cm, $\beta = 0.05$ gm cm^{2}sec, $P_0 = 1 \times 10^5$ gm cm^{-2} sec^{-2}, $\delta = 0.01$ cm and $\mu_a = 0.0002$ poise for various values of μ_m. The observation reveals that air and mucus flow rates decrease with increase in mucus viscosity. These results are in line with the results of Verma et al [23–25].

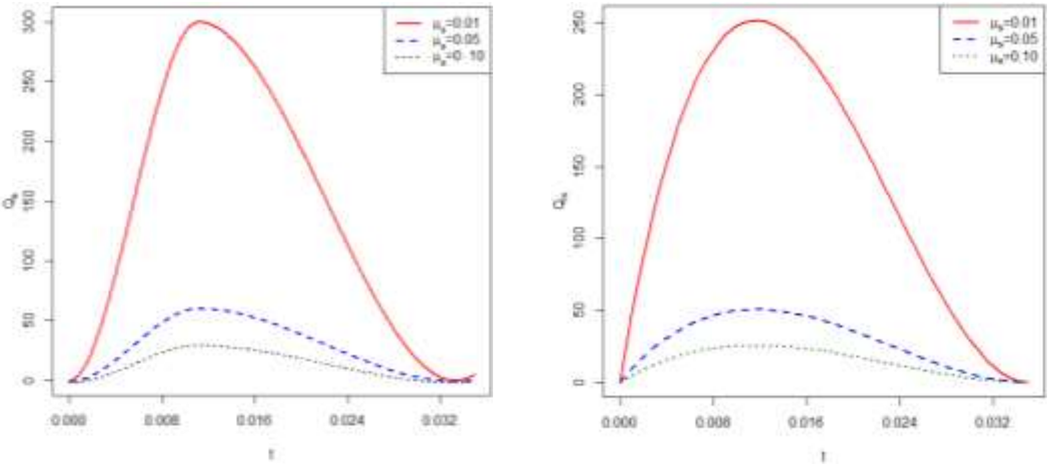


Figure 5: Variations of Q_a and Q_m with t for different values of μ_s

Figure 5 show the impact of time on air and mucus flow rates for fixed values of $T = 0.030$ sec, $L = 1$ cm, $L_0 = 0.5$ cm, $R = 90.00 \times 10^{-2}$ cm, $R_a = 31.45 \times 10^{-2}$ cm, $\beta = 0.05$ gm cm^{2}sec, $P_0 = 1 \times 10^5$ gm cm^{-2} sec^{-2}, $\delta = 0.01$ cm, $\mu_m = 1$ poise and $\mu_a = 0.0002$ poise for various values of μ_s. The observation reveals that air and mucus flow rates increase with decrease in serous viscosity. The findings are in continuation with the results of Rana et al. [13] and King et al [8–10].

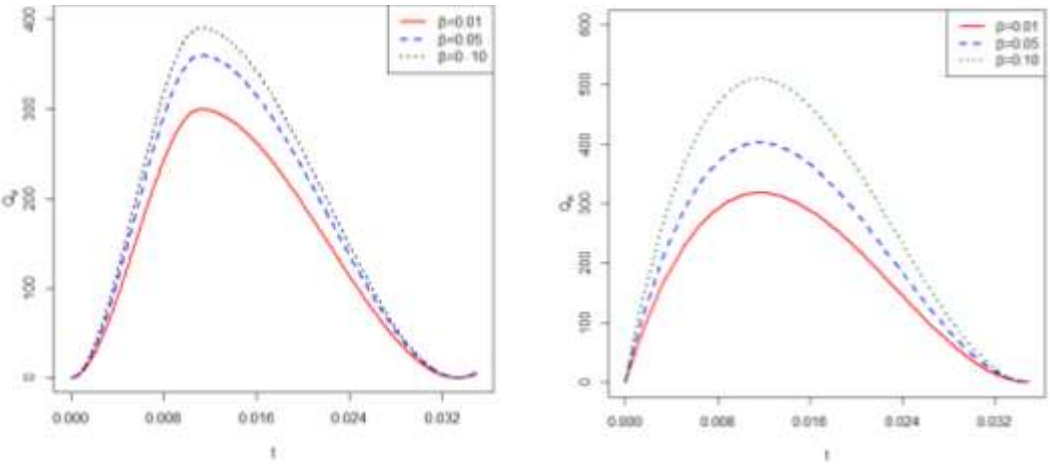


Figure 6: Variations of Q_a and Q_m with t for different values of β

Figure 6 depict the impact of time on air and mucus flow rates for fixed values of $T = 0.030$ sec, $L = 1$ cm, $L_0 = 0.5$ cm, $R = 90.00 \times 10^{-2}$ cm, $R_a = 31.45 \times 10^{-2}$ cm, $\mu_m = 1$ poise, $P_0 = 1 \times 10^5$ gm cm^{-2} sec^{-2}, $\delta = 0.01$ cm and $\mu_a = 0.0002$ poise for different values of β. It is observed that air and mucus flow rates increase as the slip parameter β increases. These results are inline with Satpathi et al. [15] and others.

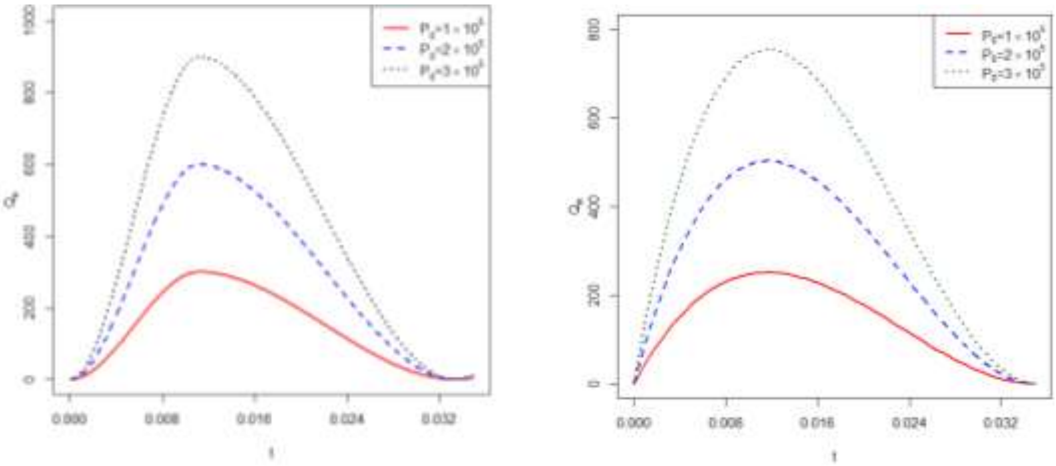


Figure 7: Variations of Q_a and Q_m with t for different values of P_0

Figure 7 depict the impact of time on air and mucus flow rates for fixed values of $T = 0.030$ sec, $L = 1$ cm, $L_0 = 0.5$ cm, $R = 90.00 \times 10^{-2}$ cm, $R_a = 31.45 \times 10^{-2}$ cm, $\mu_m = 1$ poise, $\beta = 0.05$ gm cm^{2}sec, $\delta = 0.01$ cm, $\mu_a = 0.0002$ poise and $\mu_m = 1$ poise for different values of P_0. These results is in line with the findings of Agarwal et al [1,2] and Clarke et al. [5,6].

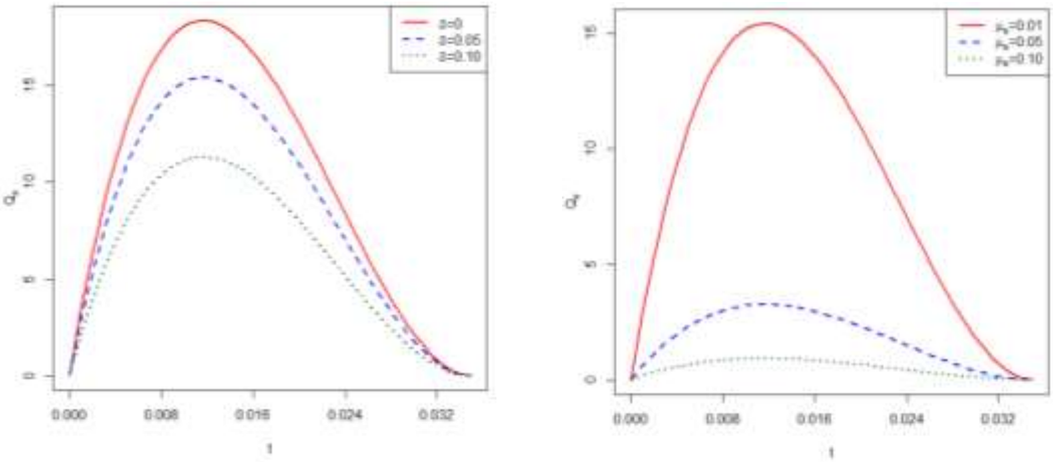


Figure 8: Variations of Q_s with t for different values of δ and μ_s

Figure 8 depict the impact of time on serous flow rates for fixed values of $T = 0.030$ sec, $L = 1$ cm, $L_0 = 0.5$ cm, $R = 90.00 \times 10^{-2}$ cm, $R_a = 31.45 \times 10^{-2}$ cm, $\mu_m = 1$ poise, $\beta = 0.05$ gm cm^{2}sec, $\mu_a = 0.0002$ poise and $\mu_m = 1$ poise for different values of δ and μ_s. It is observed that serous flow rate decreases as the constriction thickness and serous viscosity increases. These findings are in line with Sathpathi et al. [14,15] and Kumar et al. [11].

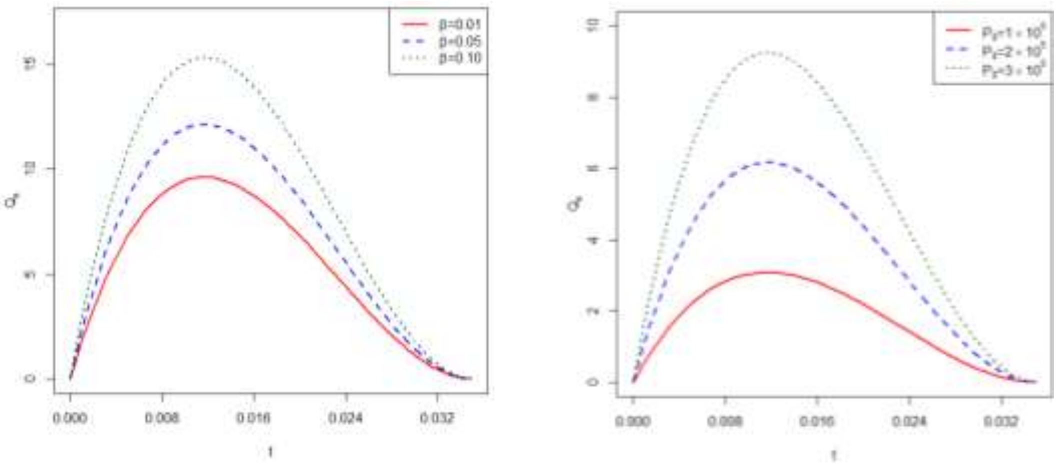


Figure 9: Variations of Q_s with t for different values of P_0 and β

Figure 9 depict the impact of time on serous flow rate for fixed values of $T = 0.030$ sec, $L = 1$ cm, $L_0 = 0.5$ cm, $R = 90.00 \times 10^{-2}$ cm, $R_a = 31.45 \times 10^{-2}$ cm, $\mu_m = 1$ poise, $\delta = 0.01$ cm, $\mu_a = 0.0002$ poise and $\mu_m = 1$ poise for different values of P_0 and β. It is observed that serous flow rate increases as the pressure drop and slip parameter increases. The findings are in continuation with the results of [1,2,15].

5. Conclusion

This study proposes a three-layer cylindrical quasi-steady coaxial flow model to analyze the stress distribution and the impact of mild coughing on the transport of air, mucus and serous fluid through narrowed lung airways. In the model, air is assumed to flow under quasi-steady turbulent conditions whereas the mucus and serous are treated as quasi-steady laminar flows. The influence of the slip parameter is also incorporated through the boundary conditions. Based on analytical and graphical evaluations, the following findings are obtained:

a) Increasing the thickness of the airway constriction leads to a reduction in the flow rates of air, mucus, and serous fluid.

b) Decreasing the viscosity of mucus enhances the flow rates of air, mucus and serous.

c) An increase in the slip parameter results in higher flow rates of air, mucus and serous.

d) A higher-pressure gradient reflecting a more intense cough which increases the flow rates of air, mucus and serous.

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